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THREE-DIMENSIONAL FRACTURE ANALYSIS

Edited By

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THREE-DIMENSIONAL FRACTURE ANALYSIS

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INTRODUCTION

A three day workshop was held at Battelle's Columbus Laboratories on April 26-28, with the purpose of bringing together the nation's leading investigators of three-dimensional fracture to discuss together the present status of three-dimentional fracture analysis. The Workshop was organized and conducted by Battelle under the joint sponsorship of the U. S. Air Force and the Department of Transportation, with contributing support of the U. S. Army Research Office.

The Workshop was organized in six sessions with the first four devoted to technical papers and discussions, one session in which government representatives discussed their problems and one session devoted to the definition of benchmark problems.

This report presents a summary of the Workshop. The four paper sessions are presented in the form of extended abstracts written by each author with summaries of discussions after each abstract. The remarks of the government representatives are summarized together with discussions of each presentation. A summary is also given of the session in which standards for benchmark problems were discussed and three benchmark problems were chosen. These problems are described in detail.

SUMMARY AND CONCLUSIONS

In organizing this Workshop, Battelle made an intensive effort to identify all of the scientists (exclusive of graduate students) in the United States who had been conducting significant research into the analysis of three-dimensional stress states around cracks. Although it is possible that some such scientists were not identified, no one has been brought to our attention as of the date of this report. Remarkably, all scientists identified as working on three-dimensional aspects of fracture accepted Battelle's invitation and attended the Workshop.

A similar effort was made to identify and invite those government representatives interested in the subject. Because of unavoidable conflicts, four of these representatives could not attend. However, nearly all of the govenment agencies interested in fracture were represented. Thus, this Workshop represented a unique event in the history of fracture analysis in terms of defining precisely the current state of three-dimensional fracture analysis and defining future directions for research.

ACKNOWLEDGEMENTS

We would like to gratefully acknowledge the support of the U. S. Air Force, the Transportation Systems Center of the Department of Transportation, and the U. S. Army Research Office without which this Workshop would have been impossible. We would also like to acknowledge the help of the Battelle participants who helped run the Workshop and prepare this report. These were: E. F. Rybicki (Session I), M. F. Kanninen (Session II), A. T. Hopper (Session III), S. G. Sampath (Session IV), T. J. Johns (Session V), L. E. Hulbert (Session VI).

SESSION I
FINITE ELEMENT ANALYSIS METHODS

CHAIRMAN, S. G. SAMPATH

CRACK TIP PLASTICITY CALCULATIONS WITH QUADRATIC "QUARTER POINT" FINITE ELEMENTS

by

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The dominant plastic singularity at the tip of a crack for a pure power law hardening material has been expressed as:

$$\sigma_{ij} \rightarrow r^{-\left(\frac{1}{n+1}\right)} \hat{\sigma}_{ij}(\theta)$$

$$\varepsilon_{ij} \rightarrow r^{-\left(\frac{1}{n+1}\right)} \hat{\varepsilon}_{ij}(\theta) \tag{1}$$

where (r, θ) are polar coordinates centered at the crack tip, σ_{ij} and ε_{ij} are the near field stress and strain respectively, n is the hardening exponent chosen to represent the experimentally determined stress-strain curve, and $\hat{\sigma}_{ij}$ and $\hat{\varepsilon}_{ij}$ are functions giving the θ dependence. The explicit determination of the $\hat{\sigma}_{ij}(\theta)$ and $\hat{\varepsilon}_{ij}(\theta)$ terms involves the solution of a nonlinear, fourth order ordinary differential equation for single mode problems in plain stress or plane strain. Mixed mode solutions become even more complex.

Efforts to characterize this singularity with finite elements have involved (1) defining the region around the crack with a high concentration of finite elements or (2) developing special crack tip elements that incorporate the singularity of Equation (1). Both "displacement" and "hybrid" formulations have been successful.

The quadratic isoparametric finite element has proven itself very valuable in two dimensional fracture mechanics problems. This is because a $\frac{1}{\sqrt{r}}$ singularity can be established in the strain displacement matrix by moving selected midside nodes to the element "quarter points". This idea has been extended to three-dimensional problems using quadratic brick and wedge type elements and again moving midside nodes to quadratic point positions. Reported here is the use of this technique on elastic-plastic problems.

Considering the "strain energy" (i.e. $\int \sigma_{ij}\varepsilon_{ij}\, dv$) near the crack tip and the near field representation of stress and strain as given by Equation 1, one may conclude that the strain energy for plastic (as well as elastic) behavior has 1/r characteristic singularity at a crack tip, i.e.,

$$\text{S.E.} = \int \frac{1}{r}\, \hat{\sigma}_{ij}\, (A)\hat{\varepsilon}_{ij}\, (\theta) dv \quad . \tag{2}$$

The stiffness matrix, K, used in finite element calculations is formed from the strain energy terms and may be expressed as

$$K = \int B^T DB\, dv \qquad , \tag{3}$$

where B is the matrix transforming displacements to strains and D is a material matrix. The quarter point adjustment has the effect of establishing a 1/r singularity in the B matrix, thus

$$K = \int \frac{1}{r}\, \hat{B}^T D\hat{B}\, dv \qquad . \tag{4}$$

The above heuristic argument thus establishes the quarter point element as a possible method for nonlinear crack tip computations in both two and three dimensions.

Comparisons have been made on a two dimensional problem using this technique and a solution incorporating a special plastic singular element. This comparison indicates that, for the case of two dimensional single mode problems, the quarter point calculations give very reasonable results.

As previously stated, this approach to crack tip plasticity is directly applicable to three-dimensional calculations. In fact, any three-dimensional elasto-plastic computer program that incorporates a quadratic element can be used as it stands, to solve crack tip plasticity problems.

Discussion

P. D. Hilton pointed out that agreement between the results presented and those of reference solutions was good on a gross behavior scale, but asked whether any comparisons on a more local scale, such as a J-integral, had been made. S. E. Benzley replied that to date those types of comparisons had not been made.

C. C. Chamis questioned the meaning of "engineering solution" as described in the presentation. Chamis contended that engineering solutions should be compared with experimental data rather than other numerical solutions.

R. S. Barsoum asked what results had been obtained for cases with flaw curvature. Benzley replied that limited work on that problem did not produce the good correlation that was presented and results near the free edge gave the least correlation.

J. Swedlow pointed out that crude analyses are sensitive to any curvature of the three-dimensional crack surface.

S. G. Sampath inquired if interelement compatibility was satisfied near the crack tip. Benzley replied that compatible elements were used in all of the results presented. Chamis inquired how equilibrium was satisfied. Benzley replied equilibrium was satisfied approximately by the finite element method.

THREE-DIMENSIONAL FINITE ELEMENT ANALYSIS FOR THE THROUGH CRACK PROBLEM

by

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The problem of a through crack in a plate subjected to in-plane loading has traditionally been treated using the two-dimensional idealizations of plane stress or plane strain. When significant yielding occurs, the observed response exhibits some three-dimensional characteristics in conjunction with stable crack growth, i.e., crack front curvature and the development of shear lips. Our long range goal is to model this problem and analyze these effects. Some initial work in the area of three-dimensional elastic finite element analysis has been performed. The three-dimensional elastic-plastic effort is underway.

Elastic Singularity Elements

Based on previous finite element accomplishments in two-dimensional fracture mechanics, a number of approaches for the development of three-dimensional singular elements for crack problems are available. We have chosen to incorporate the concepts utilized in Benzley's "enriched" elements* as the basis of a three-dimensional singular element. These elements are characterized by an assumed form for the displacement field which contains the first term of the asymptotic solution (corresponding to the square root singularity in the near tip strain field) and the standard polynomial terms associated with isoparametric elements. The stress intensity factor(s) is treated as an additional unknown (generalized nodal displacement

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* S. E. Benzley, "Representation of Singularities with Isoparametric Finite Elements", Int. J. Num. Meth. Eng., Vol. 8, pp 537-545 (1974).

component) to be determined by the finite element analysis. For the three-dimensional analog, variation of the stress intensity factor(s) along the crack front is approximated in a piece-wise fashion by polynomials, i.e., a polynomial distribution is assumed over the edge of each singular element which coincides with a portion of the crack front. The values of the stress intensity factor(s) at nodes along the crack fron are unknowns determined by the minimization of the potential energy (as approximated by the finite element method).

The use of these enriched elements which contain both the singularity solution and the nonsingular polynomial approximations for the displacement field is particularly suited for application to three-dimensional crack problems. These elements are able to model both the interior region, where the singular solution dominates in the vicinity of the crack edge, and the region near the free-surface in which the stress intensity factor may be small or zero and the large (but nonsingular) out-of-plane shearing stresses become important.

Initial results indicate that the predictions for the variation of the stress intensity factor along the crack front can be sensitive to the radius of the singular elements and that corresponding care must be taken in the development of grid patterns to obtain consistent values. Calculations have been performed which demonstrate the influence of plate thickness and of crack front geometry on the stress intensity factor distributions.

Preliminary Elastic-Plastic Calculations

A three-dimensional, incremental theory plasticity, finite element computer program has been developed to study influences of plasticity for the through crack problem (At present, no special treatment of the crack edge behavior is incorporated.). Preliminary results indicate that the tensile stress (σ_y) ahead of the crack edge varies more rapidly along that edge when plasticity is accounted for. In particular, elastic-plastic predictions for σ_y are larger than elastic predictions in the vicinity of the mid-plane of the plate and smaller than elastic predictions near the free surface. This result is consistent with experimental observations on crack front curvature.

Further calculations modeling increments of crack front growth are planned in an effort to understand the conditions for stability of this growth and the corresponding development of both crack front curvature and shear lips.

Discussion

Question: Why was K_I at the free surface set to zero in the finite element calculations? Motivation for doing this was questioned on the basis that K_I at the free surface is not known. Hilton showed results indicating that when K_I at the free surface was not present, the resulting K_I at the center changed very little. Another discussion topic was that the three-dimensional result was 10 percent higher at the center of the plate than the result for the two-dimensional case. The effects of crack front curvature and tunneling were brought up as being important in affecting the distribution of K_I values. A question was asked concerning attempts to find a crack front shape that produced a constant value of K through the thickness. Hilton answered that preliminary attempts to do this have not been successful.

QUARTER POINT SINGULARITY ELEMENTS AND THEIR APPLICATION TO THREE-DIMENSIONAL FRACTURE ANALYSIS

by

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Special quarter point singularity finite elements are applied to the elastic fracture analysis of three-dimensional surface and through cracks. The applicability of the elements to the elastic-plastic fracture analysis is also discussed.

Recently, it was discovered by the author[1,2] and independently by Henshell and Shaw[3] that second order isoparametric element formulation allows a $1/\sqrt{r}$ strain singularity when the side nodes are placed at the 1/4 point position. This singularity is exactly that of the elastic crack solution. Due to the existence of constant strain terms, rigid body motion and compatibility with surrounding elements, they converge to the exact solution and admit easily loadings such as thermal loadings. It was shown that extremely high accuracy in stress prediction close to the crack tip can be obtained using a reasonable size mesh (4). The same theory was also applied to the 8-noded isoparametric thick shell element in the case of a through crack in a plate subject to bending[5]. The comparison of this analysis with three-dimensional analytical solutions[5] lead to extremely high accuracy since the formulation of these elements is based on Reissner's plate theory.

Quarter point three-dimensional elements were applied to the analysis of a semi-elliptical crack subject to thermal shock.[6] There is no analytical solution for the thermal shock problem, however, comparisons with results obtained by the alternating method technique for the case of axial loads were in good agreement.

In evaluating the accuracy of the results of the quarter point elements, it was found that the triangular form of these elements leads to far superior results than the rectangular form[4,7,8]. This led to further investigation of the element singularity[8,9]. It was found[9] that the triangular form of the elements contain both $1/\sqrt{r}$ singularity and $1/r$

singularity. The $1/\sqrt{r}$ singularity is achieved when the crack tip is not allowed to blunt, while the $1/r$ singularity is achieved when the crack tip is allowed to blunt by having multiple independent nodes at the tip. Tests on power law hardening materials showed that the stress distribution at the integration points is in good agreement with other results obtained using special power law crack tip elements[10]. The only problem remaining with the three-dimensional elastic-plastic analysis of cracks is the large computer time required. This is due to the fact that fracture analysis requires a rather refined mesh with large number of degrees of freedom in order to obtain a meaningful answer. At the moment this problem remains to be resolved.

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ON A 3-D "SINGULARITY-ELEMENT" FOR COMPUTATION OF MIXED-MODE STRESS INTENSITIES

by

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Two topics will be briefly discussed in the presentation. The first deals with a finite-element procedure for the calculation of combined modes I, II, and III stress intensity factors, which vary, along an arbitrarily curved three-dimensional crack front in a structural component. The finite-element model is based on a modified variational principle of potential energy with relaxed continuity requirements for displacements at the interelement boundary. The variational principle is a three-field principle, with the arbitrary interior displacements for the element, interelement boundary displacements, and element boundary tractions as variables. The unknowns in the final algebraic system of equations, in the present displacement hybrid finite-element model, are the nodal displacements and the three elastic stress-intensity factors at nodes along the crack front. Interelement displacement compatibility is satisfied, by assuming an indenpendent interelement boundary displacement field, and using a Lagrange Multiplier technique to enforce such interelement compatibility. These Lagrange Multipliers, which are physically the boundary tractions, are assumed from an equilibrated stress field derived from three-dimensional Beltrami (or Maxwell-Morera) stress functions that are complete. However, considerable care should be exercised in the use of these stress functions such that the stresses produced by any of these stress function components are not linearly dependent. Since the method is based on a rigorous variational principle, which enforces at least on an average the conditions of interelement displacement and traction continuity when $\sqrt{r}$ type displacements are included in the near-tip region, the convergence of the finite-element solution for nodal displacements as well as the stress-intensity factors is established mathematically. The geometry of the "basic element" used presently, is a 20 node isoparametric "brick" element, with 60 degrees of freedom per element. The relevant matrices are evaluated numerically, using non-product type quadrature formulae with proper mathematical transformations being used when singular-type functions are encountered in stresses and strains in the near-tip region.

The ease with which the above 'singular' element can be implemented in existing general purpose, efficient, 3-D finite element codes will be discussed.

The second topic deals with research in progress on the application of the "edge-function" method to 3-D crack problems. The edge-function method may be described as a piecing together of "asymptotic" solutions to the governing differential equations for the several parts of a domain to satisfy the boundary conditions in a discrete least squares sense. The edge-function method for the present problem leads to a 'super-element' with embedded singularities similar to the one described above.

Some sample problems to check the developed procedures will be discussed. One of these is the problem of a through the thickness crack in a finite-width plate subjected to combined tension, in-plane shear and out-of-plane shear. In this problem, the boundary layer effects at the intersection of the through crack with the surface of the plate will be discussed. The second problem is that of a sandwich plate with a part-through and a debonding crack. It is assumed that two outer layers are bonded through an adhesive of constant thickness, and one of the outerplates is assumed to contain a through crack of finite length. The results obtained for this problem are compared with the two-dimensional analysis results of Erdogan and Arin. Another set of problems to be discussed is that of circular or elliptical shaped cracks that are either embedded or at the corner of a tension bar of square cross section. The obtained results are compared with those of Cruse using a boundary-integral method.

ACKNOWLEDGEMENTS

This work was partially supported by the AFOSR under grant 74-2667.

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DISCUSSION

The discussion following this presentation focused on the question of whether the three stress intensity factors are sufficient to explain all of the crack tip behavior and the dependence of K values on the technique used to evaluate K. It was pointed out by G. Sih that results at the free surface suggest the K_I, K_{II}, and K_{III} may not be enough to describe the behavior and this question should not be dismissed.

COMMENTS ON 3D FINITE-ELEMENT ELASTIC CRACK ANALYSIS

by

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The discussion concerns the writer's experience in adapting the finite-element method to 3D elastic crack analysis. The work described was performed to satisfy the need for an accurate means to apply LEFM concepts to complex structures, and also to provide a way to accurately evaluate specimens used for crack growth and fracture testing purposes. The choice of the finite-element method to address these problems naturally stems from the basic versatility of the method: the most complex loading involving the most awkward geometry can be modeled in a systematic, standardized fashion. We will be concerned here with those features of a finite-element model which influence the accuracy of crack solutions. Primary attention is given to the proper choice of deformation modes of elements used at the crack front, recognizing the singular nature of the deformation there. The necessity of precision in modeling the actual hardware load state and constitutive behavior is emphasized, as this dictates the degree to which the advances in finite-element formulation are practically beneficial.

Analytical studies have revealed that the inverse square root singularity governs at a 3D crack front, and, excluding a Mode III crack face sliding situation, in an asymptotic sense a plane strain deformation state exists at the front. (One important qualification to this is at the point where a crack front intersects a boundary, firm knowledge of the singularity there is lacking.) Experience with element assumed fields of the standard polynomial type has supported what theoretically has been predicted: convergence to an accurate singularity solution cannot be achieved with non-singular deformation representations. This experience has led to the invention of numerous techniques for deduction of the stress intensity factor(s) from fundamentally inaccurate crack solutions. The discussion here will not elaborate on these techniques, but instead, will address the use of singularity elements to achieve accurate crack solutions, from which unambiguously follows the stress intensity factor distribution or any other field quantity.

While a plane strain square root singularity is known to asymptotically exist along a crack front, there appears to be no additional mathematical insight that can be exploited for developing a formulation generally applicable to buried, surface, and through crack problems. It cannot be determined a priori over what extent the singularity dominates, and thus, what size elements should be used at the front. Hence, strictly speaking a convergence study should be a part of any crack analysis. Whereas for 2D problems, elements which contain higher order terms of the crack tip expansion can be used as an alternative to very small tip elements which represent only the leading singular term, this is not possible for 3D problems, since the 2D plane strain crack tip expansion only holds very close to the front. The encouraging fact is that accurate solutions to important test problems have been obtained using singularity elements with quite manageable meshes, indicating the viability of the approach for general applications.

The problems discussed are those described in the publications: Int'l. Jour. Fracture, 9, pp 340-343, 1973, and Nuc. Engr. and Design, 26, pp 282-290, 1974. In this work the assumed displacement method was used with six node wedge shaped singularity elements surrounding the crack front. The interpolation function had displacements in planes normal to the front. depending upon the square root of distance from the front and depending upon the angular and front direction coordinates in a regular fashion. The displacement component along the front was given a form with non-singular gradients, and thus the element was designed for a combined Mode I-II situation. As in standard finite-element formulations, the unknowns were nodal displacement values. The singular element has the necessary rigid translation modes. It does not have the linear displacement mode which would represent rigid body rotation or constant strain (such as for average thermal expansion). Clearly, the significance of a linear displacement mode relative to the square root mode diminishes as the front is approached (except for the trivial cases where $K_I = K_{II} = 0$, as in unconstrained uniform thermal expansion). Therefore, an accurate convergent solution is attainable with the element by successively decreasing element size. Practically speaking, however, this matter of linear displacement mode can be a serious constraint on element size and must be considered on a case by case basis.

The test problems are the buried penny crack, the semi-circular surface crack, the quarter-circular corner crack, and the compact tension specimen. The K_I distributions are described, and also the general features of the complete solutions. Comparisons with other published works suggest that the singularity element solutions are highly accurate. With the wide range of applicability of the singularity element approach and demonstrated accuracy, it is felt that great progress has been made in our capability to analyze 3D cracked hardware.

DISCUSSION

The discussion following this presentation was concerned with the powers of r used in the finite-element solution and element sizes near the crack tip. The term $r^{1/2}$ was included, but a procedure for including r^{α}, where α would be determined by the program, was not part of the study. Tracey reported that the decreasing element size gave a worse solution. This was attributed to the area over which the $r^{1/2}$ term was active.

AN EVALUATION OF THE QUADRATIC ISOPARAMETRIC SINGULARITY ELEMENT

by

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Concern for a reliable technique to determine stress intensity factors for both two-dimensional and three-dimensional crack problems led the author to an evaluation of the available crack tip elements. In late 1974 and early 1975 the author became aware of the potential of the quadratic isoparametric 8-node planar and 20-node solid finite-elements as singular elements[1,2]. The evaluation of the two-dimensional 8-node quadratic isoparametric singularity was made during 1975[3] and presented at the Ninth National Symposium on Fracture Mechanics in Pittsburgh on August 8, 1975. An evaluation of the convergence characteristics of the three-dimensional 20-node quadratic isoparametric singularity brick element is presented, as well as a brief study of the 8-node axisymmetric singular element.

The compact tension specimen was chosen as the example problem to evaluate the 20-node singular element. Several investigators have analyzed this Standard ASTM E399-74 fracture specimen[4,5,6,7]. Based solely on these references, it is not clear to the author that convergence of the respective solutions in these references has been demonstrated. Three grid refinements of 64, 216, and 312 elements were selected to study the convergence. The mid-surface stress intensity was determined using the displacement extrapolation method with the vertical displacements along the free surface of the crack. Table 1 presents the results for both the three-dimensional condition and the plane strain condition, in which displacements perpendicular to the specimen faces were set to zero. The 312 element grid gave a stress intensity factor to within 1 percent of the accepted plane strain, K_{2D} value; while the three-dimensional condition gave a midplane stress intensity factor 7.6 percent greater than the accepted K_{2D} value. Earlier finite difference results of Ayres[8] for a center through-thickness cracked plate suggest an elevation of stress intensity at midplane of 10 percent above K_{2D}. More recent work by Schroedl and Smith[9] using the photoelastic stress freezing technique, gave calculated midplane stress intensity factors of from 8 to 10 percent above K_{2D}. The compact tension problem was rerun with wedge elements surrounding the crack front and, while the extrapolation value of stress intensity did not change, the stress intensity determined from the quarter-point node displacement on the crack for the plane strain case increased from 7,008 ksi $\sqrt{\text{in.}}$ to 7,134 ksi $\sqrt{\text{in.}}$ giving a 0.9 percent difference compared to the K_{2D} value found by Brown and Srawley. For the three-dimensional condition with 318 elements (wedges around the crack), this difference was found to be 7.8 percent higher than the K_{2D} result for the midplane value of the stress intensity.

In addition to the above study, the 8-node axisymmetric singular element was evaluated. Two axisymmetric geometries were run: a solid cylinder with a penny-shaped crack and a cylindrical shell with a circumferential flaw. The solid cylinder was run with a uniform axial load. The cylindrical shell was run with both a remote uniform load and a non-uniform load acting over the crack. Two crack sizes (a/t) were run for both load conditions. Comparison of stress intensity factors with accepted solutions from the literature showed the 8-node axisymmetric singular element gives excellent results.

Based on the present evaluation, it appears that both the 20-node singular isoparametric brick element and the 8-node singular axisymmetric rectangular element are excellent singularity elements which require only a relatively few elements for obtaining accurate stress intensity factors. In addition, the non-singular form of the element is currently in most finite-element stress analysis computer programs, thereby eliminating the ncessity of adding a special crack tip element to the existing computer code.

TABLE 1. COMPACT TENSION SPECIMEN
20-NODE QUADRATIC ISOPARAMETRIC
THREE-DIMENSIONAL SINGULARITY ELEMENT

NO. OF ELEMENTS	DEGREES OF FREEDOM	K_I [1]	% DIFFERENCE *
PLANE STRAIN CONDITION			
64	1079	6950	–3.5
216	3275	7050	–2.1
312	1633 [2]	7125	–1.0
THREE - DIMENSIONAL CONDITION			
64	1200	7450	+3.5
216	3528	7650	+6.3
312	4965	7750	+7.6

1 MIDPOINT Z - DISPLACEMENT

2 ONE LAYER OF 20 - NODED ELEMENTS

* COMPARISON WITH K_{2D} = 7200 PSI IN

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DISCUSSION

The boundedness of the strain energy density was discussed. Bloom briefly showed the basis for the existence of a bounded energy. He found that triangular elements gave better results than rectangular elements, but if one is extrapolating, it does not make a big difference.

ON SOME FRACTURE ANALYSIS RESULTS AT THE BERKELEY NUCLEAR LABORATORIES

by

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This is an extemporaneous address on some investigations being carried out primarily by T. Hellen and W. Blackburn and their colleagues.

With the objective of solving three-dimensional crack problems, an initial study was carried out to carefully compare results obtained in two-dimensional problems with four approaches.

1. Standard analysis with standard and special crack-tip finite-elements
2. Solving crack problems with two slightly different length cracks
3. Virtual crack extension approach
4. J-integral approach, modified for thermal stress.

Comparisons between these methods were discussed. It appeared that special crack tip elements were necessary in most cases to obtain satisfactory solutions with all of the approaches.

One three-dimensional crack considered was the cross-corner crack in a pipe T-connection. This problem was investigated with a finite-element program with substructuring, in which a special cracked substructure was developed. Four different crack sizes were solved.

A second three-dimensional problem studied was the "pop-in" problem for compact tension specimens with "hard" loads. This problem was analyzed both for a straight-through crack and for a thumbnail crack.

DISCUSSION

There was a general discussion following the last presentation. Discussions of some topics brought up are summarized here. The question of the type of behavior in the isoparametric element with nodes at the quarter points was brought up by J. Bloom with the specific point that there are papers in the literature contending that this type of element is not correct because the strain energy is not valid. Bloom pointed out that the extrapolation procedure produces results similar to other analyses.

The question of the behavior in the compact tension specimen was discussed by T. Cruse. It appears that some data show that results converge to the plane strain value of K while others converge to 10 percent greater than that value. Various aspects of the solution technique including the type of finite-element, the variation of displacements through the thickness and the sensitivity of the solution to small changes in the loading were discussed. It was brought out that the comparisons being discussed have not been defined precisely enough to determine if the problems being solved are indeed the same ones. J. Swedlow pointed out that the sensitivity of the compact tension solution to changes in the loading requires that loadings be described in fine detail in order for valid comparisons to be made. T. Cruse said that he found a variation of 10 percent in the resulting K for a compact tension specimen just by introducing the loads in a different way.

G. Sih brought out the point that three types of numerical solutions for the value of K approaching a free surface were presented. In one case, it goes up, in another it goes up and comes down, and the the third case, it goes down. His comments were that different numerical approaches can contribute to the characteristics of the solution and that one should not discuss 5 to 10 percent differences until the techniques are defined. While it is not known what the three-dimensional solutions should be it was generally agreed that for two-dimensional solutions, correlation with plane strain results is necessary, but not sufficient for verification of an analysis technique.

Reasons for introducing a term of the type r^{α} in the solution and questions about the value of α being different on the free surface and the interior were discussed. These discussions as well as previous discussions, pointed out that the present state of knowledge about the three-dimensional solution is such that the characteristic power of r at the free surface is not known and while this question is being addressed, an exact elasticity solution does not appear to be on the immediate horizon. Concerning numerical techniques, observations that the form of the strain energy within the quarter point isoparametric elements loses it's $r^{1/2}$ characteristic were expressed. However, numerical results pointing to convergence of numerical solutions and theoretical bases for convergences were brought out to support the utility of the numerical techniques. Thus, while the three-dimensional numerical techniques are a valuable tool for the stress analysis, there are no exact three-dimensional solutions to serve as a basis for comparison. This provides a fertile ground for discussion and future research in this area.

SESSION II
CRACK TIP PHENOMENA

CHAIRMAN, A. T. HOPPER

STRESS INTENSITY ESTIMATES FOR THREE-DIMENSIONAL CRACKED BODY PROBLEMS BY THE FROZEN STRESS PHOTOELASTIC METHOD

by

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Based upon an idea of G. R. Irwin[1] and analytical studies of G. C. Sih[2] and his associates, the senior author and his associates have developed, over a period of years[3-22] a computer assisted, frozen stress photoelastic technique for estimating stress intensity factors (SIFs) in three-dimensional cracked body problems. Originally developed for Mode I problems only, the method has been extended to Mixed Mode problems (i.e., Modes I and II) and studies are currently being conducted which are directed towards the inclusion of Mode III effects in the measurements.

For Mode I SIF estimates, the analysis involves expressing the maximum shearing stress in a plane mutually perpendicular to the flaw border and the flaw surfaces in the form

$$\tau_{max} = \frac{K_I}{(8\pi r)^{1/2}} + \sum_{N=0}^{M} A_n r^{N/2} \quad , \qquad (1)^*$$

along a line normal to the flaw surfaces and passing through the crack tip. By truncating the Taylor Series to its first term only, and combining the result with the stress-optic law in two dimensions

$$\tau_{max} = \frac{n'f}{2t'} \quad , \qquad (2)$$

where n' is the photoelastic fringe order, f is the material fringe value and t' the slice thickness, an expression of the form

$$\frac{K_{Ap}}{\bar{\sigma}(\pi a)^{1/2}} = \frac{K_I}{\bar{\sigma}(\pi a)^{1/2}} + \frac{A_o(8)^{1/2}}{\bar{\sigma}} \left[\frac{r}{a}\right]^{1/2} \quad , \qquad (3)$$

* Where r is distance from crack tip.

can be obtained, where $K_{Ap} = \tau_{max}(8\pi r)^{1/2}$ which shows $\frac{K_{Ap}}{\bar{\sigma}(\pi a)^{1/2}}$ vs $\left[\frac{r}{a}\right]^{1/2}$ to be a linear relation. By obtaining photoelastic data and using least squares to fit a line to this data, the graph of $\frac{K_{Ap}}{\bar{\sigma}(\pi a)^{1/2}}$ vs $\left[\frac{r}{a}\right]^{1/2}$ can be extrapolated to the origin to obtain K_I. By working directly with τ_{max}, stress separation methods are avoided.

Following the ideas of Reference(5), the authors have developed a similar approach for estimating both K_I and K_{II} values in a Mixed Mode problem.

The experimental data are obtained by constructing a scale model of the body from fringe free transparent photoelastic material, inserting a "natural" or an artificial flaw, and heating the structures to the critical temperature of the material. At this temperature, the material is fully elastic, and it is loaded to a desired level. The model is then cooled under load, "fixing" both the photoelastic fringes and the deformations so that both are retained in the material even after unloading and slicing of the model. Thin slices mutually perpendicular to both the flaw border and the flaw surfaces are then removed at designated locations along the flaw border, immersed in a liquid of the same refractive index as the model material and analyzed in a crossed circular polariscope via the Tardy Method. Results are fed into a least squares digital computer program for estimating the SIF for each slice.

Figure 1 shows typical fringe patterns for both Mode I and Mixed Mode loadings and Figures 2 and 3 show typical results for two three-dimensional problems.

These results show that, even for rather complex three-dimensional problems, the linear zone can be located experimentally and results such as those shown here can be replicated to within $\pm$ 5 to 7 percent.

Since Poisson's Ratio $\simeq 0.5$ for all stress freezing materials above critical temperature, it is necessary to include a correction when it is desired to apply results to a material for which $\nu \simeq 0.3$. This correction ranges from $\simeq$ +5 percent for surface flaws to as much as 12 percent for through cracks in plates of finite thickness. Studies are in progress to accurately quantify this effect.

To date a fairly broad variety of technologically important three-dimensional problems have been studied. It is felt that the method offers a reasonable independent experimental check on finite-element results provided experimental error of the order of 5 percent can be tolerated.

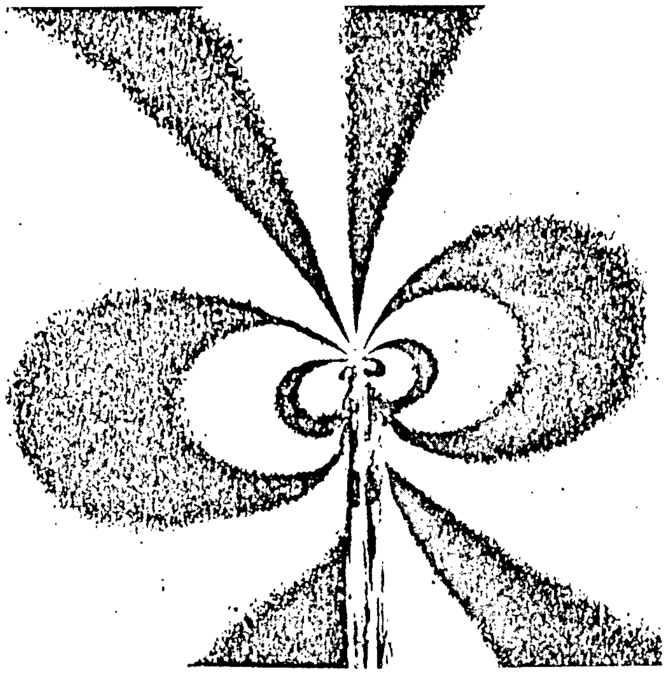

Figure 1a. Typical Fringe Pattern Near S (Figure 2)

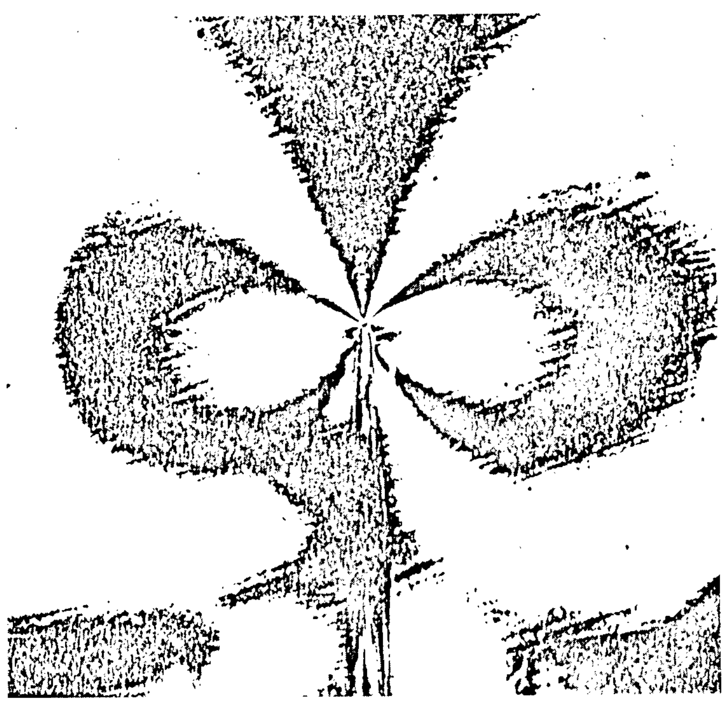

Figure 1b. Typical Fringe Pattern Near H (Figure 2)

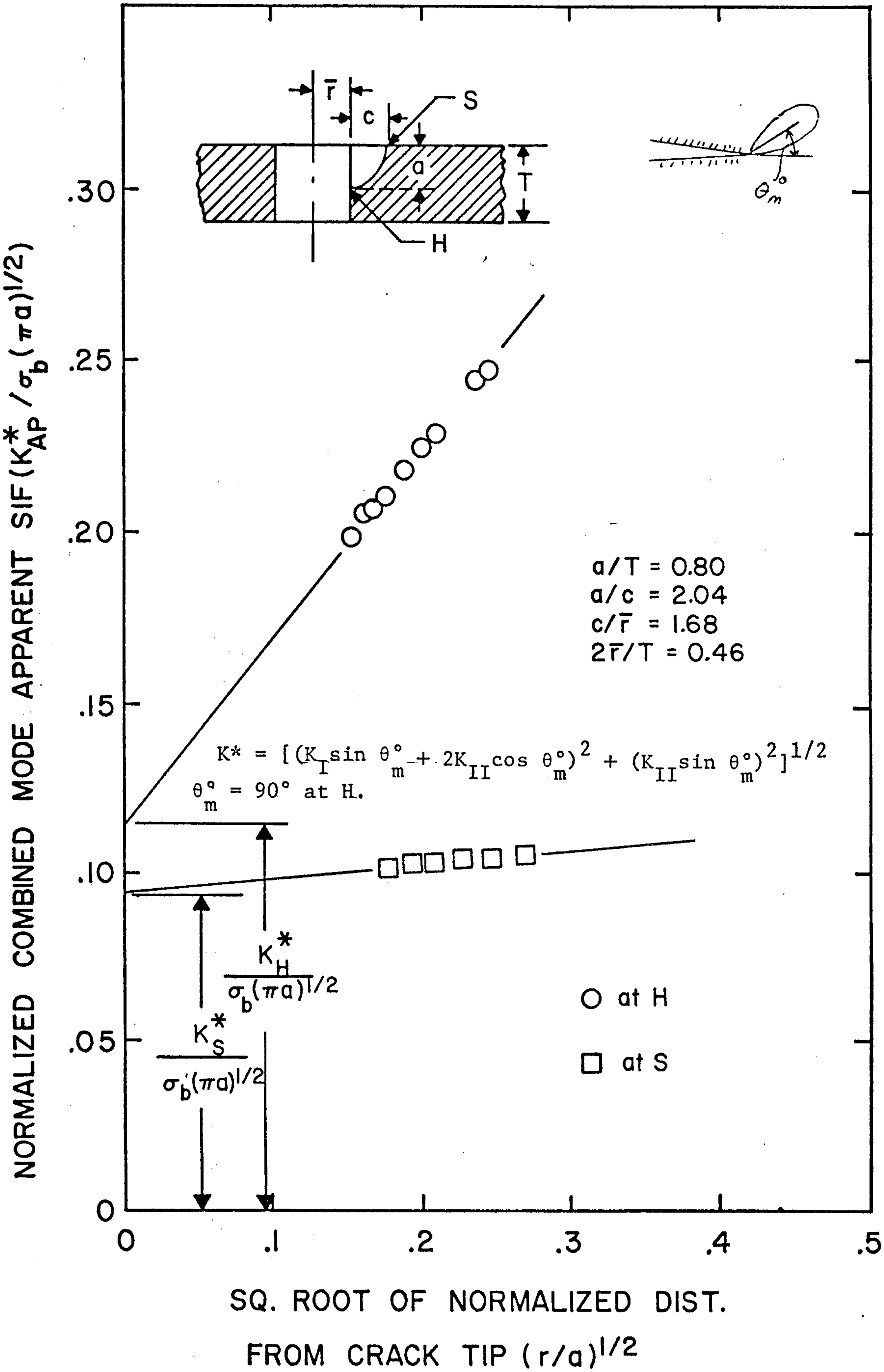

Figure 2. Typical Set of Data and K* Determination

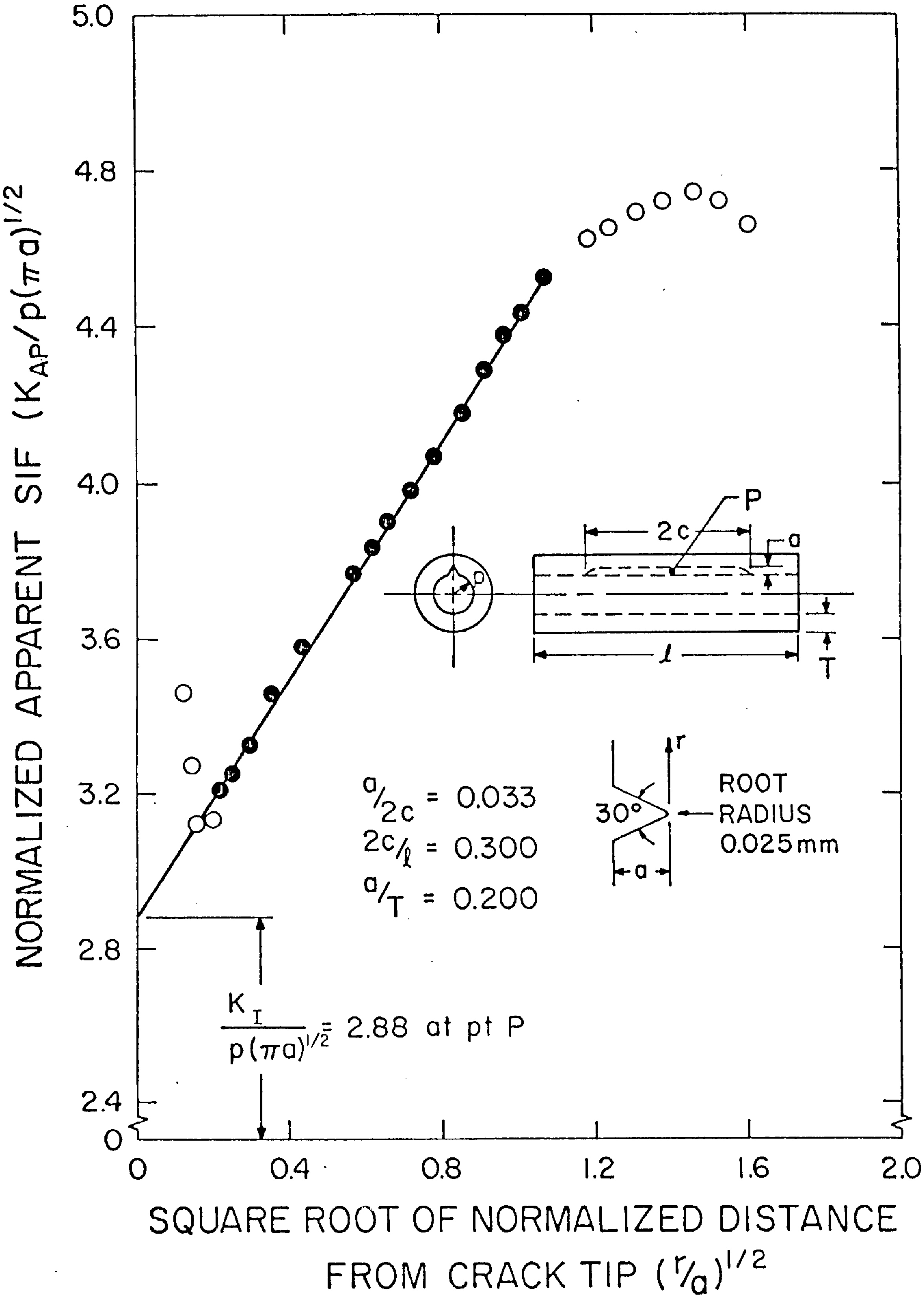

Figure 3. Typical Set of Raw Data and SIF Estimation

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DISCUSSION

Swedlow raised an important point in that most photoelastic analysis work is focused on obtaining the stress-intensity factor, but, other information can be obtained of value for checking a finite-element computation. Smith concurred, but pointed out that, while numbers such as the COD's can be obtained from a photoelasticity solution, they are not unequivocal. A correction must be introduced which depends on the modulus and Poisson's ratio of the material. While this is straightforward in two-dimensional problems, it is not in three-dimensions.

OBSERVATIONS OF CRACK TIP PROCESSES

by

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Several metallographic methods can be used to study three-dimensional crack problems in addition to the photoelastic method discussed here by Professor C. W. Smith. The Fe-3Si alloy-dislocation etching technique[1] has been used to good advantage to reveal the plastic zone of cracks. Figure 1 shows examples of the plastic zone produced by a fatigue crack both on the surface and in the interior on the midsection of a compact specimen[2]. This etching technique is specific to Fe-3Si steel, a material with a stress strain curve similar to medium strength constructional steels. It has been used to delineate crack tip plastic zone produced by monotonic loading[3], cyclic loading[2], stable crack growth[4], and fast fracture[4,5].

Three-dimensional effects have an important role in the evaluation of the material properties governing fast fracture and crack arrest. Figure 2 shows an example of the profile of an arrested fast fracture in the A533B steel test section of a duplex DCB test piece[6]. Figure 2a (about 15 mm from the crack tip) shows microscopic branch nuclei, which are believed to be responsible for the macroscopic branching event observed in a comparison specimen shown in Figure 3. The branch nucleation event is a poorly understood three-dimensional phenomena which complicates the measurement and interpretation of propagation and arrest events. Deep side grooves, which can be used to restrict branching events of this kind also introduce a three-dimensional component to the stress field of the crack. Figures 2a and 2b show ligaments, which are another three-dimensional feature of predominatly cleavage fractures[5]. There is evidence that the major part of the toughness displayed by steels tested below the transition temperature is derived from the ductile fracture of the ligaments which contributed a small fraction of the total fracture surface[5].

Figure 4 shows the arrested crack front which was revealed by heat tinting the crack surface (dark portion) before breaking the specimen apart at -196^{o} C (light portion). The light line visible behind the crack front is a long unbroken ligament (reverse tunnel) which exerted a drag on the crack front.[8] Analyses that can relate such perturbations of the crack front to local variations in the fracture resistance, would be useful.

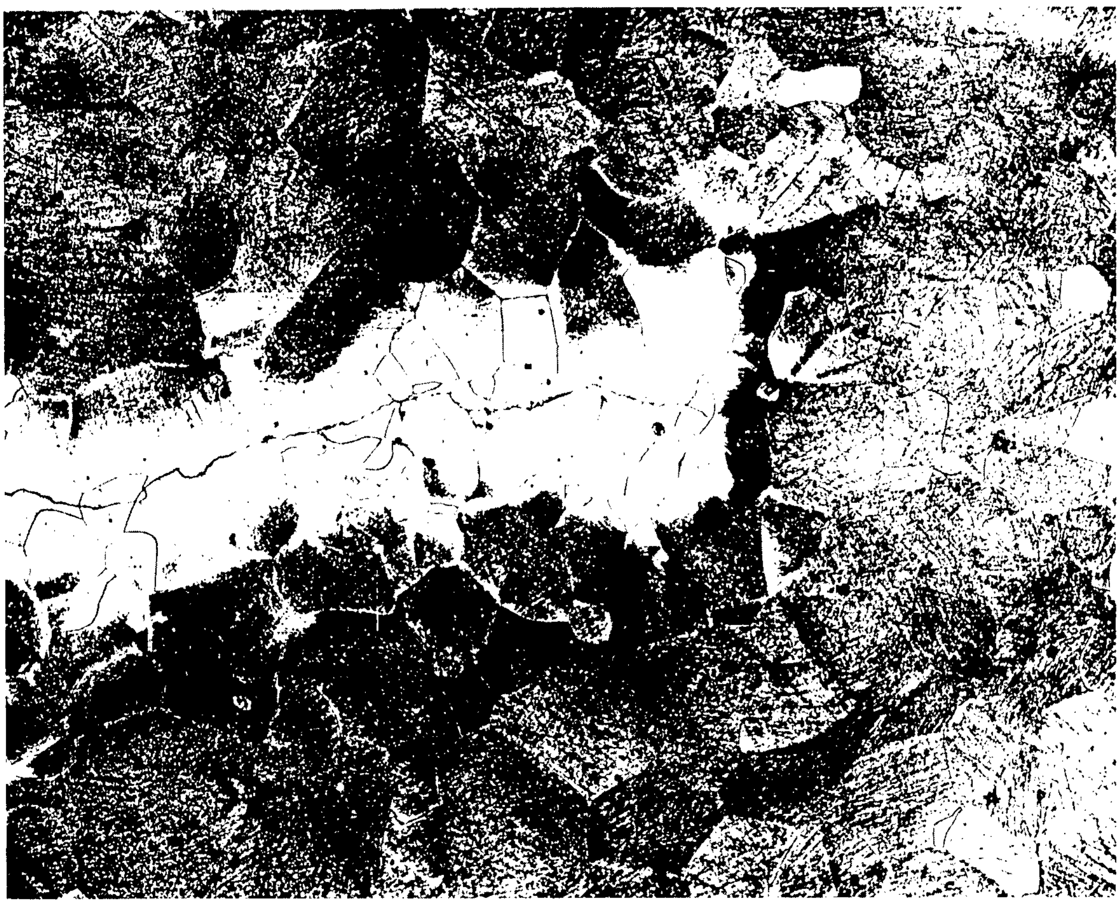

FIGURE 1. PLASTIC ZONE OF A GROWING FATIGUE CRACK REVEALED BY ETCHING(2). Magnification 500X.

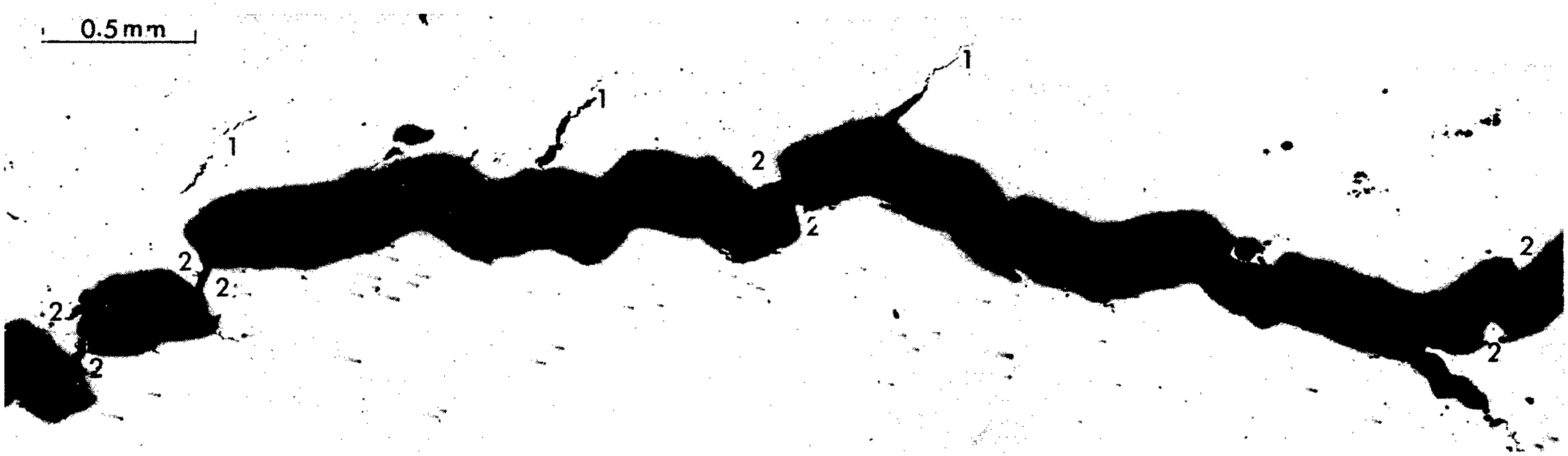

FIGURE 2. PROFILES OF A BRITTLE FRACTURE PROPAGATION IN A533B STEEL: (a) 15 mm from arrest crack tip, (b) at the crack tip. The photos show branch nuclei (1) and ligaments (2).

(b)

FIGURE 2. (Continued)

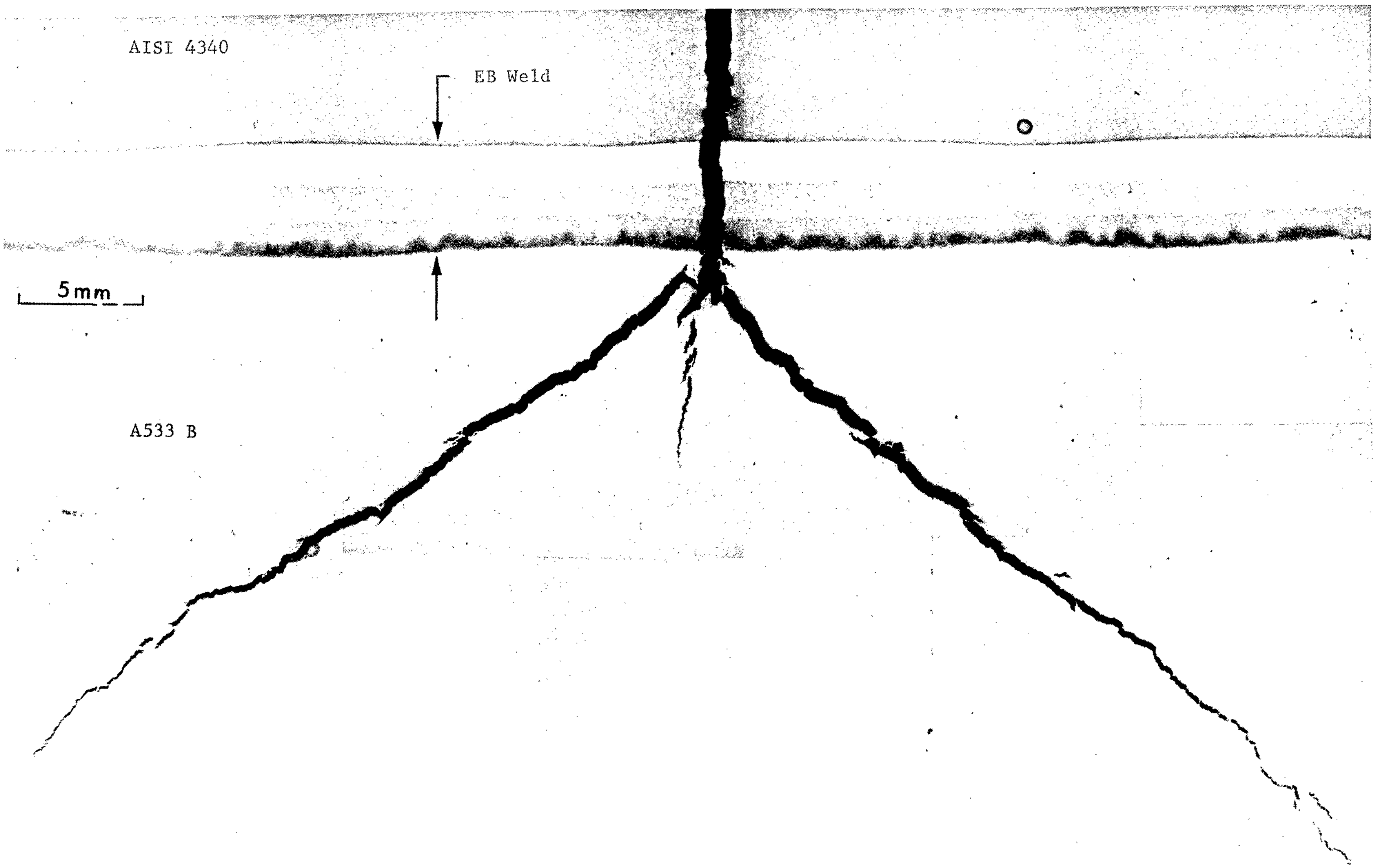

FIGURE 3. PROFILE OF BRANCHED FRACTURE ON PLATE MIDSECTION OF DUPLEX DCB SPECIMEN. The crack propagated from top to bottom and branched in the A533B steel after passing through the electron beam weld which remains intact.

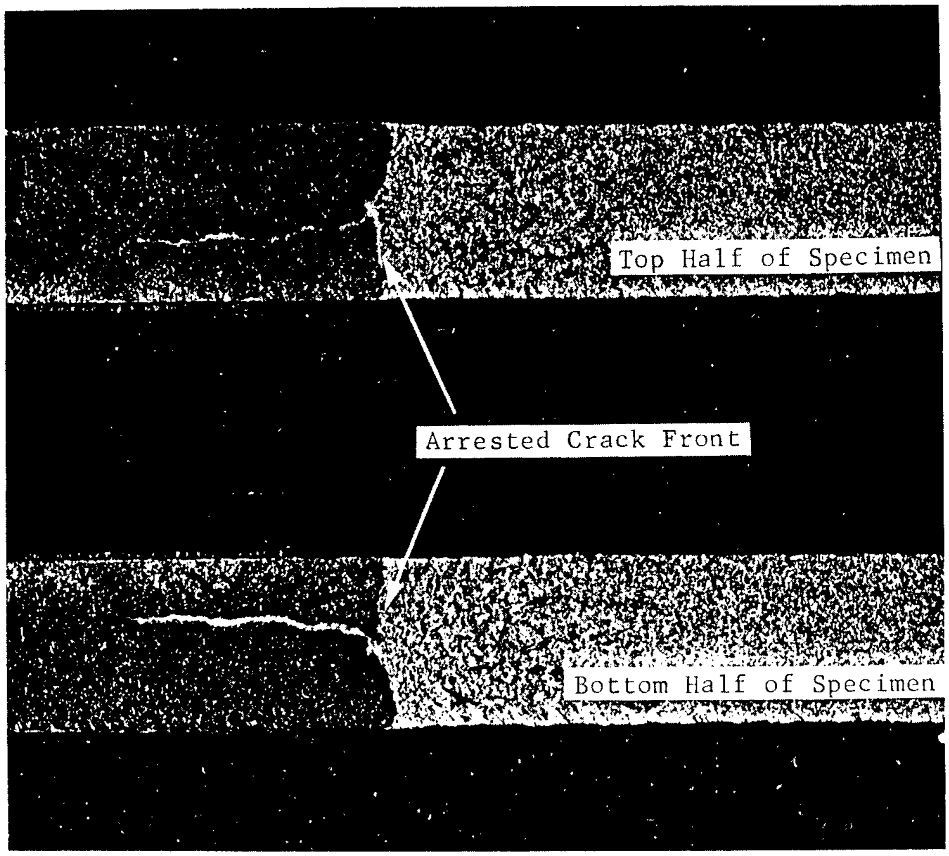

FIGURE 4. PHOTOGRAPH OF AN ARRESTED CRACK FRONT. The crack front, which propagated from left to right, was produced by wedge loading a A533B steel DCB-test piece (with deep sidegrooves) at -78 C. The front was delineated by heat tinting the surface of the arrested crack (dark area) and then breaking the specimen open (light fracture).

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DISCUSSION

Cruse initiated an extensive discussion regarding tunneling (where the crack advances faster in the inerior than on the surfaces) and reverse tunneling as a three-dimensional phenomenon. No conclusions were reached as to how it might be treated, however.

USE OF CYCLIC GROWTH TESTS TO INFER STRESS INTENSITY

by

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The technique of inferring stress intensity values from experimental crack growth tests and a knowledge of a crack growth-stress intensity relationship is increasing in popularity as a tool for treating complex geometry problems. Some caution is required to insure that the inferred stress intensity is correctly depicted and not confused by material and/or experimental variables.

For illustration, the preliminary assessment of some test data is discussed. This data came from constant amplitude crack growth tests of center cracked tension geometry specimens with various sized through holes centrally located. These specific through-crack tests were conducted to verify the experimental technique and material behavior prior to conducting tests of holes with corner cracks.

The initial data assessment showed three anomalies when inferred stress intensity values were compared to theoretical values for the CCT geometry with the Bowie treatment for near hole effects and the Fedderson secant correction for finitie width effects. For crack lengths up to twice the hole radius, there appeared to be a stress level effect wherein the lower the cyclic stress the higher the implied stress intensity factor. For long cracks, the stress intensity factors started to increase more sharply than the secant width correction would provide. And finally, there were several indications of a random shift of implied stress intensity when higher growth rates were experienced.

The short crack length anomaly is shown to be the result of a subtle mis-representation of cyclic growth rate-vs-delta K data for growth rates below 10 micro-inches per cycle.

The long crack anomaly can be numerically accommodated by arbitrarily adjusting the secant width correction by 10 percent. The only basis suggested for this arbitrary adjustment is that it reasonably fits the data for a fairly large variety of specimen widths and crack lengths greater than 50 percent of specimen width. Additionally, it was noted that a similar adjustment to the secant width correction was required to match elastic compliance data for CCT specimens with long crack lengths.

The third anomaly is attributed to the variety of three-dimensional geometries that through cracks may assume at high stress intensities and high growth rates. At the transition from flat to slant crack geometry two basic forms may occur: full slant or cup-cone. Since the center crack geometry has two crack fronts a large variety of combinations will occur such as matching or opposed slant, matching or opposed cup-cone and combinations where one crack is slant and the other is cup-cone.

Substantial variations of out of plane displacements result and the differing resultant mixed mode stress intensities are reflected in differing crack growth rates. All of which can be avoided by only conducting tests at low stress intensity values so that cracks are only grown in a flat plane normal to the principal applied stress.

One further complicating factor was illustrated by crack growth data for one aluminum alloy, single product form, single thickness. Significant variation of growth rates were observed between material from different manufacturers. A factor of two variation occurred at 10 ksi $\sqrt{in}$ and 5 microinches/cycle and more than an order of magnitude difference occurred above 20 ksi $\sqrt{in}$.

It was concluded that the technique of inferring stress intensity from cyclic growth rate tests was viable. However, considerable care is required in test conduct and reconciliation of results.

DISCUSSION

Cruse was disturbed by the fact that obvious large cyclic plasticity and the possibility of mixed modes were present in the experiments, yet comparisons were made from specimen to specimen where significant differences in these conditions were ignored. Specifically, the fatigue crack growth data involved plasticity and mixed mode mechanisms that are necessarily thickness dependent. Correlation can be expected only where the plate thickness and mode of crack growth are identical. On this basis, Cruse suggested that plasticity could play a large role in the work and that the correlation problems evidenced by Collipriest might be explained in this way.

SUMMARY OF COMMENTS FOR WORKSHOP ON THREE-DIMENSIONAL FRACTURE ANALYSIS

by

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It is evident that analytical (computational) procedures are now in hand to attack a useful range of three-dimensional elastic problems involving cracks. It is my sense, however, that individual cases selected for study are frequently chosen more on the basis of the simplicity of their geometrical and loading configuration than the utility of the end result. That is, the issues encountered in laboratory (data-collecting) and service (data-applying) situations should be more influential when problems are posed for analysis. To exemplify this view, one situation of some pertinence is reviewed.

Some experimental information[1,2] has been developed which suggests that, at least in a compact specimen, crack-tip K values are not what would be computed using standard formulae, e.g.,[3]. While the nature and scope of this work does not yet give a full picture of behavior germane to the issue, two notions emerge from even these limited data:

(1) Formulae for K pertinent to the compact specimen are especially sensitive to details of load arrangement (as opposed to overall magnitude). This point is noted elsewhere[4] and may carry over to "three-dimensional" configurations currently of popular interest, e.g., the corner crack embedded in a pressurized vessel.

(2) Configuration of the crack itself may have a disproportionate influence on local K values. An early suggestion to this effect[5] concerns a number of geometries other than a standard - or nearly so - test specimen. Some dramatic results obtained in the convenient cases were treated in[5].

The relevance of these notions to three-dimensional fracture analysis is to be noted in two quite different respects. Both show that accurate K values are not of necessity derived from planar analysis, either in some global or average sense, or along the crack front. Were such information in hand, however, interpretation of some test results would be clarified. As noted in[1], for example, the simple load-carrying capacity of a specimen derived from analyses of the form summarized in sources such as[3,4] may in some instances become misleading. The inference drawn in[5] is that interpretation of fracture test data may be nonconservative. Of perhaps greater importance is the collection and use of fatigue test data. Under cyclic loading of a CS specimen, the crack grows and the contours of its front may alter; no cognizance is taken of this matter in computing a planar ΔK, which is the sole mechanism for transferring data from the laboratory to design or service situations.

The relevance of better K values along the crack front is also seen in three-dimensional fracture analysis, the subject of this workshop. At present, there is less than a clear view of what problems need attention. Manifestly, numerical procedures have been developed to the point where a wide range of problems may be solved, but the setting of actual cases to be treated seems to be guided only incidently by physical observation. While the results reported here represent major investments of energy and intellect, there remains need for close attention to actual behavior of a series of specific shape/material/loading systems so that the computational power now available is advantageously used.

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DISCUSSION

The data of Neale, introduced by Swedlow, related the ratio of the local value to the nominal K as a function of the crack front curvature. Hahn noted that these data appear to conflict with the Barsoum-Clausing results in which positive tunneling increases the K level. Neale's data gives a linear decrease with crack fron curvature. Townley suggested that viewing the appearance of a thumbnail as a stability effect may be helpful.

CRACK TIP FIELDS IN STEADY CRACK GROWTH WITH STRAIN HARDENING

by

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Singular stress and strain fields are found at the tip of a crack growing steadily and quasi-statically into an elastic-plastic strain hardening material. The material is characterized by J_2 flow theory together with a bilinear effective stress-strain curve. Anti-plane shear, plane stress and plane strain are each considered. Numerical results are obtained for the stress and strain fields, the order of the singularity and the near tip regions of plastic loading and elastic unloading.

SOME PROPERTIES OF FINITE-ELEMENT APPROXIMATIONS OF ELLIPTIC PROBLEMS ON DOMAINS WITH CRACKS AND CORNERS

by

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A survey of some of the mathematical foundations of elliptic variational theory for two-dimensional domains with corners, including cracks, is described. The basic mission here is to describe the mathematical framework in which analyses of finite-element methods for problems in fracture mechanics must be studied. In particular, the theory of weighted Sobolev spaces is described together with appropriate imbedding theorems and an existence theorem for variational boundary-value problems set in these spaces. It is shown how these results help form the basis for obtaining error estimates for restricted classes of finite-element methods. A priori error estimates in energy norms in an L_2-norm and in the L_∞- norm are described. The study summarizes the principal results of Babuska and some very recent findings of Schatz and Whalbin.

NEAR FIELD BEHAVIOR AND CRACK GROWTH

by

George C. Sih

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Lehigh University
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A knowledge of the local three-dimensional stress field for a crack with an arbitrarily curved front is necessary for investigating the crack growth behavior. Such a stress field is referred to a system of local spherical coordinates (r, θ, ϕ) and the result can be reduced to a surprisingly simple form when the appropriate choice of coordinates (r, θ, ω) are used.

Crack growth directions for various positions along the crack front can be determined from the strain energy density fracture criterion. The mixed mode loading of a flat elliptical crack serves as one of the examples. The development of thumbnail cracks in a thick plate can also be predicted. A future application of this result to the fracture thickness problem is the interaction of a curved (thumbnail) crack front with the highly distorted or yielded zones (shear lips) near the plate surfaces. The analysis requires crack growth under mixed mode loading as the shear lips are developed on planes inclined to the plate surfaces.

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STRESS INTENSITY FACTOR MEASUREMENTS FOR CORNER CRACKED HOLES

by

A. F. Grandt, Jr.

Air Force Materials Laboratory
Wright-Patterson A. F. B., Ohio

The fatigue crack growth rate method was used to measure stress intensity factors at various points along the border of a corner crack located at the edge of hole in a plate loaded in remote tension. Cyclic extension of the corner flaws was recorded by time lapse photography in plexiglass test specimens. Since the specimens were transparent, it was possible to photograph full plan views of the part through crack and, thus, determine the variation in fatigue crack growth rate around the flaw perimeter. The measured fatigue crack growth rates were then used with the Paris relation between fatigue crack growth rate and range in stress intensity factor (previously established for the test material) to compute the cyclic range in stress intensity. The stress intensity factors at the points where the crack intersects the edge of the hole and the front surface of the plate are then compared with various analytical predictions available in the literature. Computations for other intermediate points along the crack boundary are in progress and will be discussed as available at the time of the meeting.

SESSION III
GLOBAL FUNCTION METHODS

CHAIRMAN, T. J. JOHNS

DEVELOPMENT OF PROCEDURES FOR ANALYZING STRESSES IN CRACKED BODIES OF VARIOUS SHAPES

by

J. C. Bell

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During the last few years, an effort has been made at Battelle's Columbus Laboratories to develop procedures for analyzing crack stresses in bodies involving progressively more detail of body shape. The progress that has been made is built on two basic forms of stress analysis and on a procedure for merging analyses of these forms in unified calculations. Recent efforts have centered around choosing details in the merger process so that dependable analyses will be obtained. Various aspects of the work have been sponsored by Battelle, by the Air Force Flight Dynamics Laboratory, and by the NASA, Lewis Research Center.

BASIC ANALYSES

The basic analyses designed for treating cracks is one for stresses and displacements around an arbitrarily loaded circular crack in an infinite body. Normal loads as well as radial and circumferential tangential loads are considered in terms of load coefficients which are arbitrarily definable so that they can eventually accommodate body-surface effects. The analysis for the circular crack alone leads to stress and displacement solutions expressed in terms of integrals of products of Bessel functions. These integrals appear formidable at first, but they can be reformulated so as to become readily computable in modern computers. (Thus, a set of integrals for a fixed radius and a fixed distance from the crack plane, but with about 1000 combinations of defining indices, can be computed in about one second by the CDC Cyber 73.) The stress-intensity factors of any mode along the crack front, which are useful in predicting whether further cracking will occur, are related quite simply to the crack-load coefficients. Two papers have been written on this subject, one deriving the theory (this has been submitted for publication), and one summarizing and illustrating it for simple cases (this has been accepted for publication in the Journal of Structural Division of the ASCE).

The second basic analysis determines stresses and displacements in a half-space subjected to certain elemental loads, both normal and tangential, on the surface. The elemental loads are quasi-pyramidal and are applied in overlapping fashion so that the overall load on the surface is continuous and varies linearly in both directions in each rectangle of a set covering the entire loaded region. The continuity of the applied load pattern avoids the spurious discontinuities of stress that arise in forms of analysis which use discontinuous (step function) surface loads. The expressions for the stresses and displacements in the body are simple, and the summation processes that are involved are readily adaptable for computers. The implied continuity of stresses on the surface has a potential advantage in that it allows much freedom in pointwise fitting boundary conditions on the surface and meaningful evaluation of boundary condition residual errors at nonfitted points.

MERGING OF CONTRIBUTORY ANALYSES

For analysis of stress around a circular or part-circular crack in a finite body, an analysis of the first basic kind is merged with one or more analyses of the second kind. This is accomplished numerically using a computer program constructed from the formulas of both kinds of contributory analyses, and the coordinate transformations needed to relate them to each other. In broad terms the merger is effected by using the boundary-point-least-squares technique to find a set of crack and surface load constants which lead to approximate satisfaction of a chosen set of boundary conditions. The fitting is accomplished by a single, least-squares calculation, not requiring the iteration needed by other investigators who have used the alternating technique for corresponding mergers of analyses. Once the proper load constants have been found, the program can be used to evaluate not only stress intensity factors, but also stress and displacement components anywhere in the body.

The finding of load constants is straightforward in principle, but there are many choices to be made in arranging the calculations. These include how long the series expansions of crack functions should be (this is decided in advance), how detailed the lattices used for surface-load decomposition should be, where boundary conditions should be imposed, and whether equilibrium should be enforced among the surface loads used to "free" the surfaces. Trial calculations have shown that all of these choices significantly affect the computed stress intensity factors, and since it is not feasible to pursue all of these matters into endless detail, there are many questions to be answered in considering whether the results of a given calculation may be regarded as being dependable.

Much of the ambiguity of designing calculations for specific bodies and load systems can be removed by following standardized design procedures, but the question still remains whether the chosen procedures lead to calculations in accord with the actual mechanical behavior of real bodies. In order to gain objectivity in this matter, it was decided to analyze cases for which experimental stress intensities were available and for which the choice of calculation design involves fairly stern consequences. The experimental work used as a reference showed stress intensities along the front of a circular surface crack in slabs of varying thickness.* One particularly challenging case had a crack four times as long as its depth, with the crack penetrating to a depth 0.85 times the slab thickness.

The testing of calculation design procedures was begun by analyzing cases of lesser severity (that is, crack depth to slab thickness ratio less than 0.85), but procedures apparently successful for those cases were soon found to give highly uncertain results for the most severe case. Through close examination of results from many calculations, however, several procedures have been chosen which have removed much of the uncertainty in the computed stress intensity factors. The basis for most of these procedures can be understood in terms of how individual crack functions vary on the

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* This experimental work was performed by C. W. Smith of the Virginia Polytechnic Institute under contract to the Air Force Flight Dynamics Laboratory.

crack and in the slab faces. One important principle seems to be that crack functions should be used to fit only crack-face conditions and surface loads to fit only slab-face conditions, except for the bonafide interactions between the different load systems. Achieving the right degree of independence of effects from the different load systems while performing a united least-square fit is, therefore, one of the most important features of a good calculation design procedure. This principle is especially important in the selection of boundary conditions to be applied. It is possible also to do much tailoring of the crack-function series in advance. The design of surface lattices is still subject to ambiguities, but here also, matters such as the interplay between lattice design and overall equilibrium have been considered.

An interesting feature of some of the calculated results is that they show maximum stress intensity factors at points well removed from the root of the crack. This is similar to the referenced experimental results and may help to explain the tendency of circular cracks to ellipticize as they grow.

POSSIBLE APPLICATIONS OF THIS ANALYSIS

The computer program that has been used in performing the calculations has many features which have not yet been applied. It allows for the presence of six faces, so that interactions may be analyzed between a crack and the sides and ends of the body may be considered, as well as interactions from the top and bottom faces. Tilting of the crack is allowable, and it may penetrate more than one face of the body. For consideration of a body bounded in three-dimensions, the program allows for imposition of equilibrium of overall forces and moments. Since consideration of several faces may be expected to make calculations cumbersome, the program also includes means for condensing the calculations in view of symmetries which may exist in the particular body being analyzed.

The computer program is complete also in the sense that it contains formulas for computing stresses and displacements throughout a body for which the load constants have been found. This makes it possible to study the variation of stresses in the vicinity of the crack, a possibility that may help in understanding other forms of stress analysis there. Computations of displacements also offer an alternative approach to finding experimental checks of the analysis. One region in which detailed study could be of particular interest is around the end of the crack. Some calculations have shown stress intensities there near zero, but further refinements there seem to be needed.

It was planned originally that the analysis of stresses from loads on a half-space might be applied to a considerable variety of bodies with surfaces approximatable by sections of planes. This could include shapes obtainable by joining plates or by extrusion and could include curved junctions by using several planar strips successively inclined at relatively small angles, though some supporting analysis would be desirable to smooth the joining of analyses. In another vein, the basic analyses mentioned here can be, and to some extent have been, used as a basis for constructing a super finite-element including a crack. The analytic nature of the solutions presuming only crack and surface load functions lend themselves to such a development.

DISCUSSION

Fred Smith commented that the stress on the crack circle exterior to the surface must be such that the crack does not close.

C. W. Smith noted that in his experimental work, slices taken on opposite sides of the crack center line give the high/low effects at successive points in his K vs angle curves.

R. Shaw noted that "skinny" ellipse results show an increase in the "hump" over the results shown here.

BOUNDARY-INTEGRAL EQUATION ANALYSIS OF SURFACE CRACKS*

by

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East Hartford, Connecticut

Introduction

Elliptical surface and corner cracks growing in mode I symmetric loading are a major low cycle fatigue (LCF) design problem for gas turbine engine structures. During the majority of the LCF life of such cracked components these nearly-elliptical cracks maintain modest aspect ratios (semi-axis ratio <3) and see no significant "back-face", finite geometry effects. In order to predict the growth of such cracks it is necessary to predict the mode I stress intensity factor along the entire crack perimeter, denoted K(S). The numerical results presented herein reflect a systematic development of a modeling strategy for such crack problems. Numerical modeling was accomplished using the BIE three-dimensional stress analysis computer program described in Reference 1.

Modeling Strategy

The use of a general stress analysis method for fracture mechanics problems is not as accurate as the use of analysis methods with "crack tip" elements, or other singular techniques. In three-dimensional fracture mechanics problems in LCF, two measures of accuracy need be considered. First is the absolute magnitude of the stress intensity factor at some crack front location; second is the distribution of K(S) along the crack front.

* Research reviewed in this presentation is being sponsored by the Air Force Office of Scientific Research (AFSC), United States Air Force, under Contract F44620-74-C-0060 with Mr. W. J. Walker (NA) as Air Force Program Manager.

BIE-generated numerical results obtained for elliptical buried cracks have been reported for a series of aspect ratios ($a/b \leq 4$) and are given in Reference 2. The BIE element map used in these studies is shown in Figure 1. Values of stress intensity factor at increments of 15 degrees elliptical angle are estimated from the crack opening displacements of the row of nodes nearest the crack tip. These values of K(S) have been found to be in error by about 8-12 percent, as reported in Reference 2. However, by dividing the K(S) values for the ellipse models by the values for the circle, this systematic modeling error was essentially canceled. Figure 2 compares the resulting values of normalized K(S), i.e., the K-distribution, to the exact results for the buried crack.

The excellent agreement achieved for buried cracks using this procedure has been used for fracture mechanics results in surface and corner crack problems. For example, a 2:1 corner crack K-distribution is obtained by dividing the crack opening displacements for the corner crack by the values for the buried crack of the same aspect ratio (for data reduction simplicity). The actual values of K(S) are obtained by multiplying the normalized, numerical results by the exact values of K(S) for the buried crack.

Figures 3 and 4 are normalized values of K(S) for surface and corner elliptical cracks under uniform tension transverse to the crack plane. Three elements combine to obtain K at any location: 1) the front face or free surface effect on K at the surface; 2) the free surface effect on K at the deepest location; and 3) the effect of crack shape on K(S). The third effect is partially contained in the normalization term; its influence on the normalized data in Figures 3, 4 is secondary. In general, the results show that the free surface magnification is around 30-35 percent at the surface for both problems; the magnification at the deep location is about 5 percent for surface cracks and 12 percent for corner cracks. Further numerical data are given in Reference 3.

Current numerical modeling is focused on evaluating the effects of two other geometrical variables: finite thickness and the angles at the crack-free surface intersection. Correlation of the numerical results for surface and corner cracks is being generated by comparison of predicted fatigue crack growth to experimental results. Results to date show good correlation at both near-surface and deep crack locations.

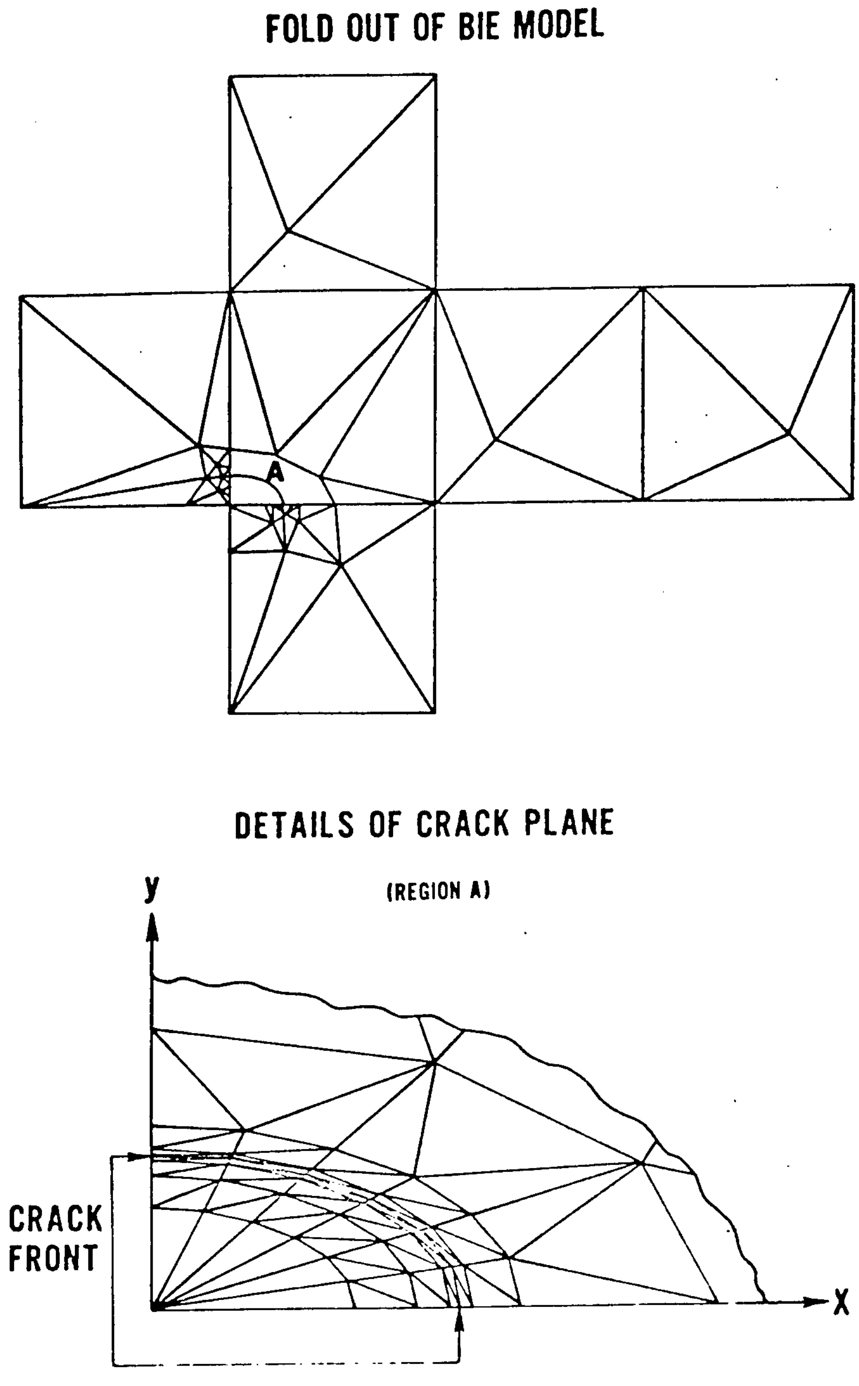

FIGURE 1. BIE MODEL FOR ELLIPTICAL CRACK PROBLEM

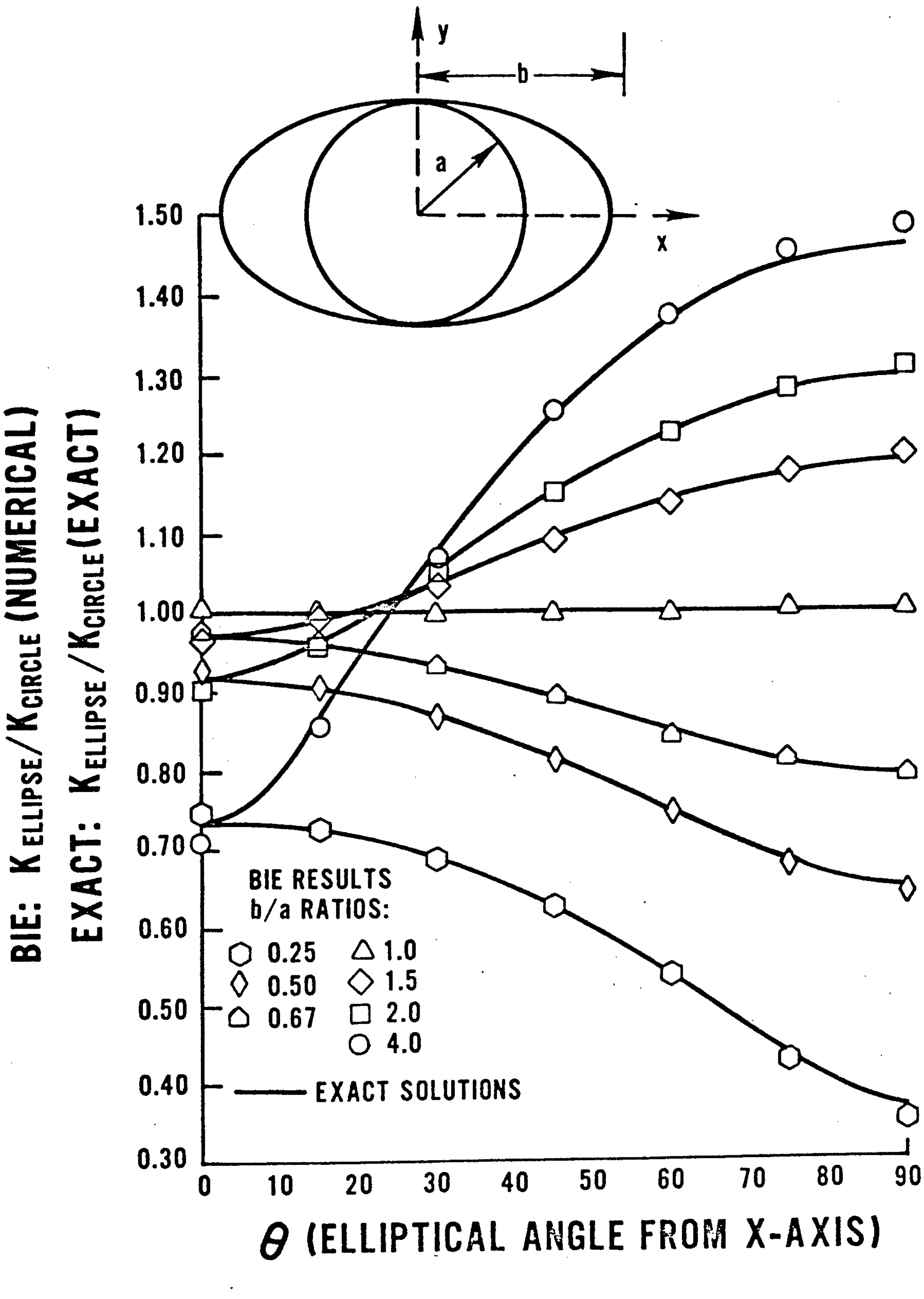

FIGURE 2. VARIATION OF STRESS INTENSITY FOR BURIED CRACK

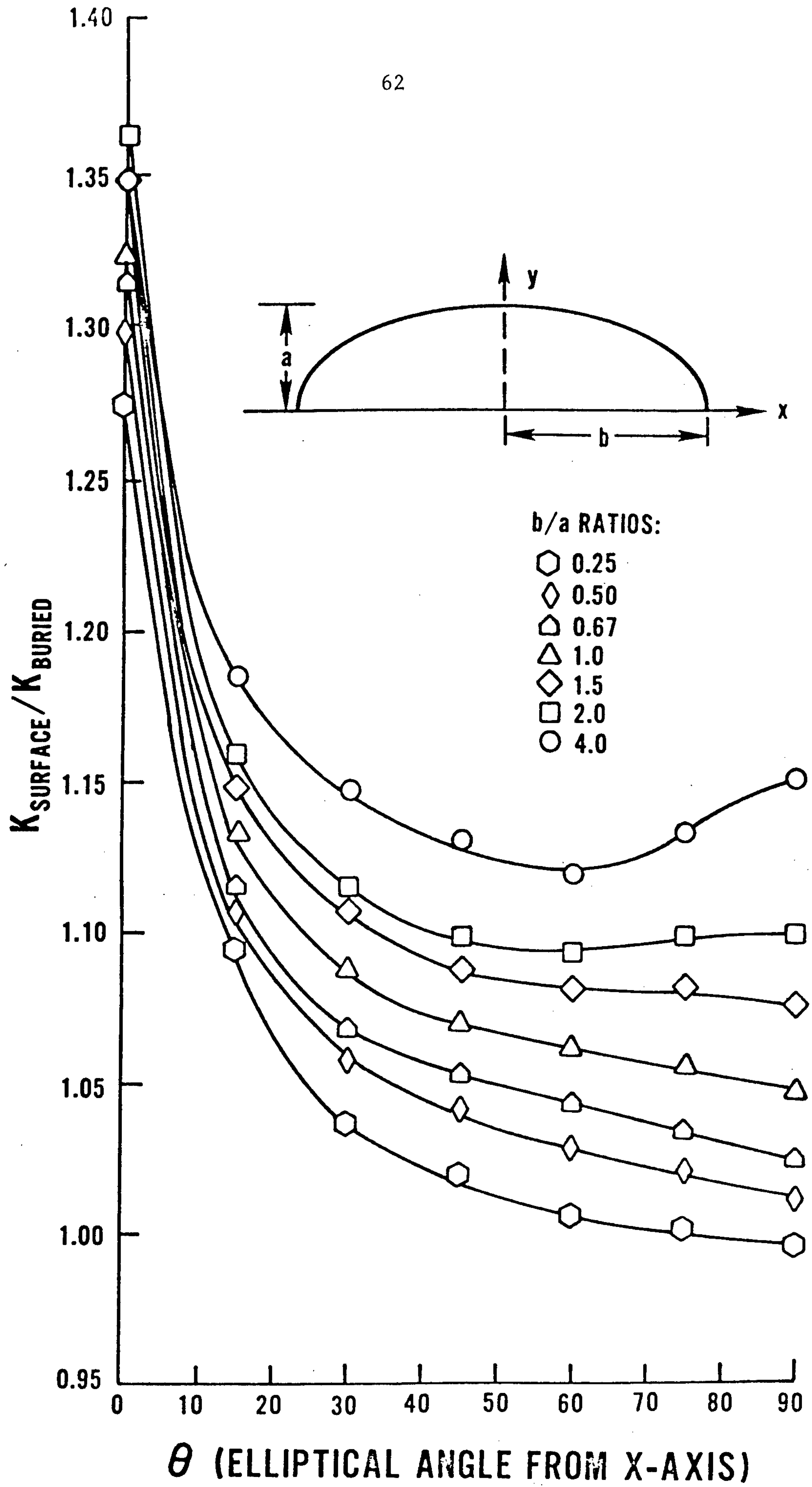

FIGURE 3. STRESS INTENSITY MAGNIFICATION FOR SURFACE CRACK

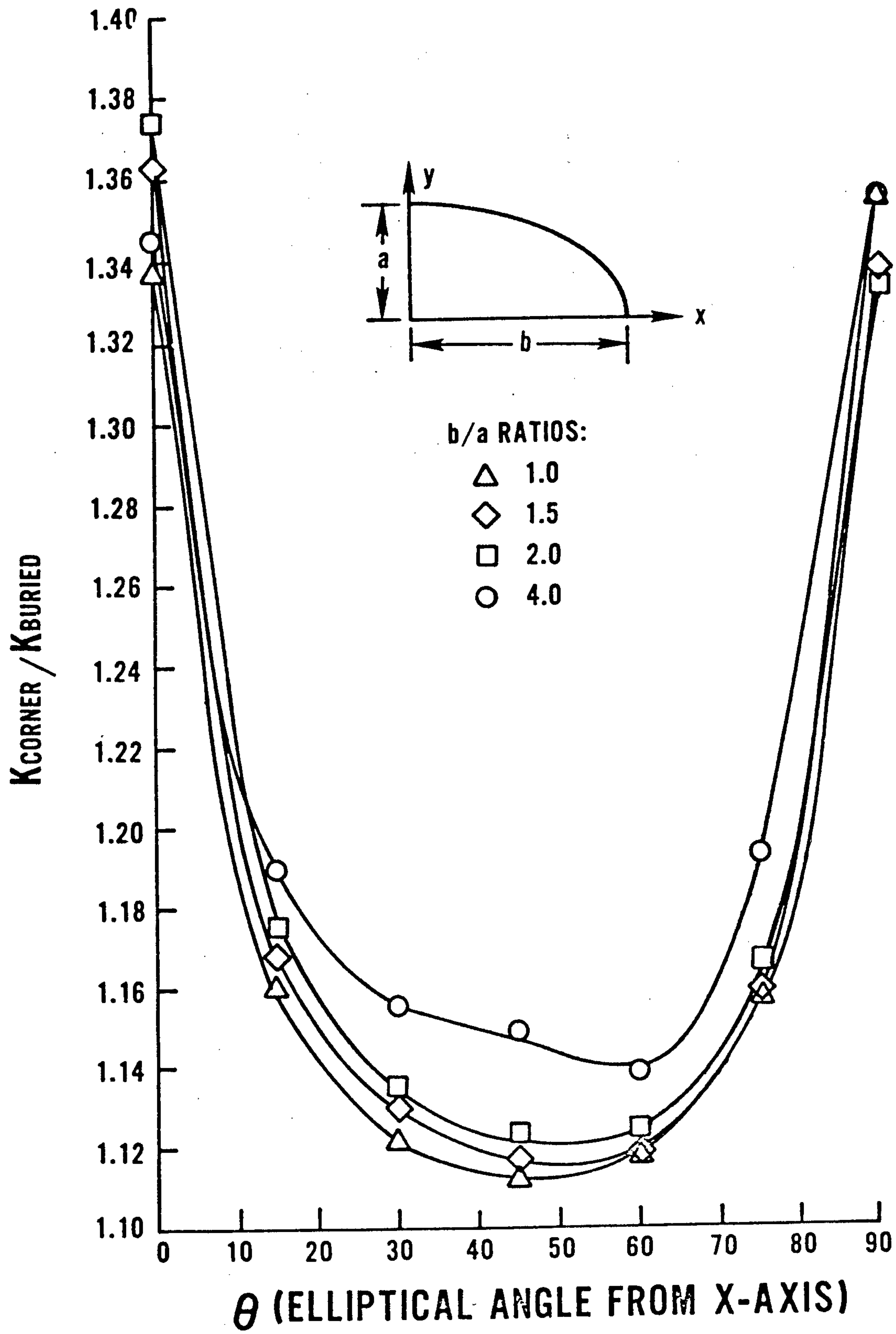

FIGURE 4. STRESS INTENSITY MAGNIFICATION FOR CORNER CRACK

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DISCUSSION

Question: You said ligament stresses, if anything, go down with increasing crack depth. How is that so?

Cruse: It is due to the way the crack sheds load.

BIE computed stress intensity results are believed to be too high. I believe the experimental results.

The stress intensity, by definition, does not go to zero at the free surface. However, whatever goes on at the free surface does not affect the crack at the root.

Question: How was K calculated?

Cruse: From C.O.D. next to the crack tip. It was not extrapolated but a normalization procedure was used.

As the crack gets very deep, K seems to tail off as C. W. Smith and J. C. Bell have experienced. However, I was not willing to go beyond a/t of 0.7 because of the dimpling.

Bell: With regard to the back surface effect, there are no singularities at the back surface. So what happens is that as complications occur on the back surface, they influence the crack stress which in turn influences the front surface even more. Thus, in the end, the front face is still the most critical area for my analysis.

Cruse: My analysis shows that the back face stress intensity magnification factors are most dominatly tied to the K at the front face. Neighboring effects are most critical for elliptical cracks. The most crude local information is obtained in the back face model so that I have trouble correlating deep crack K's with experiment.

SUBSURFACE ELLIPTICAL FLAWS

by

A. S. Kobayashi, N. Polvanich, A. F. Emery,
and W. J. Love

The well-documented alternating technique in three-dimensional fracture mechanics was used to obtain our recent results on stress intensity factors of:

(1) An elliptical crack near or partially penetrated through a re-entry corner of a three-quarter infinite solid subjected to a polynomial distribution of stresses;

(2) An elliptical crack near a square corner of a quarter-infinite solid subjected to a polynomial distribution of stresses;

(3) A large elliptical crack in a thin plate subjected to a polynomial distribution of stresses.

Two iterations of the alternating technique yielded in ninety percent of the crack surface residual surface tractions less than one percent of the maximum prescribed pressure for the largest proximity ratio of $b/h = 0.9$. Residual surface tractions on the two bounding flat surfaces of the solid were less than one percent of the maximum prescribed pressure. Numerical convergence for the partially penetrated elliptical crack at a re-entry corner was enhanced by prescribing appropriate fictitious pressure on the penetrated portion of an elliptical crack. The results are presented in terms of a stress intensity magnification factor, which is the actual stress intensity factor divided by the corresponding stress intensity factor of an embedded illiptical crack in an infinite solid, for constant through cubic terms of polynomial crack pressure distribution.

For an embedded elliptical crack of crack aspect ratio of $b/a = 0.02$ and $a/h = 0.8$ and at a square corner, the second bounding surface effectively increases the stress intensity magnification factor, M_K by only 0.65 percent over the corresponding solution of an embedded elliptical crack in a semi-infinite solid[1]. For $b/a = 0.2$ and $a/h = 0.9$, distinct influence of the second bounding surface is seen by a rapid increase in the stress intensity magnification factors up to 16 percent. For a 20 percent partially penetrated crack at a re-entry corner, the stress intensity magnification factor increases to about 1.11 near the free surface in contrast to the 1.23 values observed in semi-circular surface problems.[2]

Published results on inner cracks in thermally shocked cylinder[3], unpressurized inner[3] and outer cracks[4] in pressurized cylinders, pressurized inner semi-elliptical cracks in pressurized cylinders[4] as well as unpublished results involving a comparison of calculated and experimental crack mouth openings in surface flawed plates subjected to tension and bending[5] are also presented.

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DISCUSSION

F. W. Smith: For an ellipse in the middle of a plate, we also showed that interaction is important. I agree with your statement on bending moments far away.

Kobayashi: I should have run a larger area and not tried to save on computing time.

In the surface flaw, cross coupling plays a large effect. The model should have been taken all the way to the grips.

STRESS INTENSITY FACTORS FOR A PRESSURIZED THICK-WALL CYLINDER WITH A PART-THROUGH CIRCULAR SURFACE FLAW - COMPLIANCE CALIBRATION AND COLLOCATION METHOD

by

M. A. Hussain

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Watervliet, New York

The K calibration at the deepest point of a circular surface flaw in an internally pressurized thick-walled cylinder has been determined by a compliance test as well as three-dimensional collocation method. Compliance is defined as the change in internal volume of a cylinder divided by applied hydrostatic pressure instead of usual elongation/load definition. It is shown, using linear theory of elasticity that the derivative with respect to crack depth of internal and external volume changes are identical. This permits the use of external strain measurements to compute the stress intensity factors. Periodic cubic splines are used to interpolate/approximate the strain data as well as to compute external volume change as a function of crack depth.

With a starter surface notch of semi-circular cross-section, an extensive set of tests were performed on a cylinder with 7.1 inch smooth bore and 14.25 inch outside diameter. A fatigue crack was grown from this starter notch until it reached each successive desired depth. The circumferential strains were then read on outside circumference at 14 angular locations. The stress intensity factors were computed by compliance technique.

The results at the deepest points compared well with exterior collocation method. In this method we represent the shape of such a crack by a segment of a circle. The objective is to construct a set of functions such that the most stringent boundary conditions in the plane of crack are satisfied exactly. These conditions are: the normal displacement u_z is zero outside the circle and the normal stress σ_z is zero inside the circle. For a symmetric problem, the shearing stresses vanish in the entire crack plane.

Using integral forms of harmonic functions, the above conditions are satisfied and the rest of the boundary conditions exterior to the plane of the crack are then satisfied by collocation in the least square sense.

This is analogous to the two-dimensional problem where William's stress functions have been used successfully. However, as opposed to two-dimensions, the three-dimensional surface flaw problems require an additional condition to be satisfied; namely the boundary traction must vanish at points of crack-free surface intersection. In our analysis this requirement yields a zero stress intensity factor at such points. However, the singularity at crack-free surface intersection points has yet to be investigated.

DISCUSSION

Question: Do the results represent one point or an average of the K's taken around the crack front?

Husain: The results are taken only at $z = 0$.

Cruse: Are not the strain gages on the mid crack line measuring an average SIF while your analytical data is single valued?

Husain: The strain gages are not measuring an average.

F. W. Smith: Olavi in about 1966 published results for a circular crack problem for which the surface intersects off-center and obtained different results.

ON THE THREE-DIMENSIONAL THEORY OF FRACTURE

by

E. S. Folias

University of Utah
Salt Lake City, Utah

The speaker discussed his recent theoretical results on the linear elastic, three-dimensional, stress distribution in a plate of finite thickness and containing a plane (rectangular), through-the-thickness, crack and under the action of Mode I loading.

Emphasis was placed on the physical interpretation of the results and their implications to fracture.

Finally, the results were compared with currently existing in the literature theoretical and experimental evidence and attention was drawn to some possible pitfalls.

DISCUSSION

Cruse: It is my contention that all analytical studies break down at the surface corner of the through crack because the discontinuity of the behavior is built in. Thus, you cannot get unique results.

Folias: Sternburg & Sadowsky get K going down at the corner for different geometries than I have, but these are not comparable.

Sih: How do you define uniqueness?

Folias: Wilcox's proof is short and I can send it to anyone interested. He has worked extensively with the defraction equation and was the first one to prove uniqueness for this problem.

SOME UNSOLVED SINGULARITY PROBLEMS

by

M. L. Williams

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A key to understanding fracture problems is to understand the nature of the singularity existing in the field. Consideration of some of the following unsolved singularity problems could lead to increased understanding of the singularities in 3-d fracture.

(1) A conical hole in a half space into which a rigid sphere is pressed. The problem of whether there is a singularity at the tip of the conical hole has not been resolved mathematically although there appears to be no singularity.

(2) The mathematically important question of completeness for the stress functions at a V-notch in a plane stress specimen has not been solved. This question also has not been answered for fracture in adhesive bonded joints.

(3) The problem of the oscillation of the singularity near the crack tip for mixed media (adhesive) problems makes the choice of the proper size of the finite-elements in this crack problem very difficult.

(4) A problem posed by Rongved of a square or rectangular block bonded to a half space with a normal tensile load applied to the block. What is the singularity at the corners where both the geometrical and mathematical boundary conditions are not smooth?

(5) In a center cracked plate of finite width what is the behavior as the crack nears the edge (The last fiber problem)?

(6) What is the behavior in a clamped free right angled corner as the geometry passes through the transition from no initial crack to a finite length crack along the clamped edge.

SESSION IV
FINITE-ELEMENT AND FINITE DIFFERENCE METHODS

CHAIRMAN, E. F. RYBICKI

FUNDAMENTAL STUDY OF CRACK INITIATION AND PROPAGATION

by

Mark L. Wilkins

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The program was started at the Lawrence Livermore Laboratory to predict the capabilities and potential failure of a nuclear reactor pressure vessel. A coupled calculational and experimental approach is used to predict initial failure. The work includes calibrating a fracture model in two- and three-dimensional time dependent computer programs used to simulate engineering tests of material. Examples show application of the three-dimension HEMP program to calculate the critical state of failure of a flat tensile specimen and the calculated stress field around a through-the-thickness crack in a pipe.

APPLICATION OF AN INFLUENCE FUNCTION METHOD FOR THREE-DIMENSIONAL ELASTIC ANALYSIS OF CRACKS

by

Philip M. Besuner

Failure Analysis Associates
Palo Alto, California

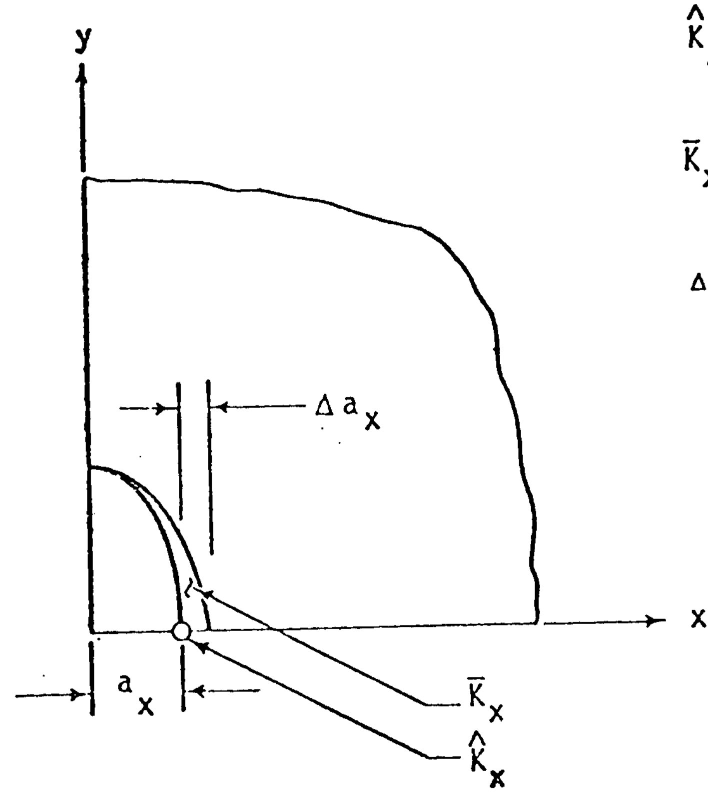
A weight or influence function (IF) method has been extended to allow accurate and extremely inexpensive calculations of elastic stress intensity factors in three-dimensional crack problems. The stress intensity factors are not local values, $\hat{K}(s)$ at some position on the crack front s, but rather are local root-mean-square values, $\bar{K}$. The $\bar{K}$ calculations are inexpensive numerical integrations of the product of predetermined influence functions and uncracked stress functions (i.e., the stress field at the crack locus in the uncracked solid). The method allows any number, n, of these rms stress intensity factor values, $\bar{K}_i$; i = 1, n to be calculated around the crack front. In the limit of asymptotically large n, it can be shown that $\bar{K}(s)$ approaches the local value $\hat{K}(s)$. In this method, the crack is described by any number of scalar dimensions a_i; i = 1, n or degrees of freedom (DOF), and each $\bar{K}_i$ corresponds to the strain energy release rate obtained by perturbation of a single DOF while holding all other DOFs constant.

The advantages of the method derive from the fact that influence functions may be calculated through the crack opening displacements determined analytically or experimentally for the desired crack geometry and displacement constraints under very simple loading, such as uniform pressure inside the crack. These influence functions may then be used to compute $\bar{K}_i$ for a general bivariate uncracked stress field, such as $\sigma_z(x,y)$. Thus, the method has the important advantage that neither error nor cost of solution is affected by the complexity of the known uncracked stress field.

The author presented justification of the use of $\bar{K}$ and summarized many successful engineering applications of the IF method in which one to four DOFs have been used to model the three-dimensional crack. Most of the applications have involved elliptical cracks with multiple degrees of freedom under fatigue loading. By computing the growth rate of each dimension a_i

from its associated $\bar{K}_i$, the method has the capability to predict non-self-similar fatigue crack growth, as is indicated in Figure 1, which illustrates a two DOF quarter-elliptical corner crack.

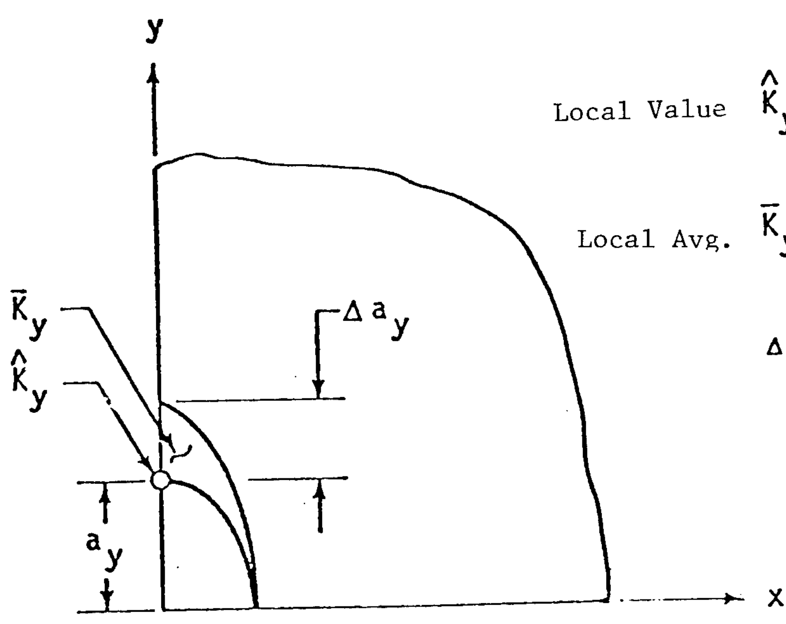
An application to the analysis of boiling water reactor feedwater nozzle corner cracks was outlined. Both the IF method and a readily available, but not optimum, finite-element (FE) method were applied to several three-dimensional crack analysis test cases. The test cases had been chosen for their similarity to a nozzle crack analysis being funded by the Electric Power Research Institute and for the availability of an accurate published result obtained from some recognized third method of solution. Results were given which summarize both accuracy and computer cost of the two methods for each of the selected cases. The IF method was demonstrated to be superior from both an accuracy and cost viewpoint. The FE method, as applied, was more than 1,000 times more costly than the IF method and had average errors of 3-5 percent and peak errors of 8 percent, as compared to less than 2 percent error for the IF method. On the basis of these results, the IF method has been chosen by Electric Power Research Institute to perform all stress intensity factor evaluations of nozzle flaws in the remaining phases of a current industry study.



$$\hat{K}_x = K\,(a_x,\ 0)$$

$$\bar{K}_x{}^2 = \frac{1}{\Delta A_x}\iint_{\Delta A_x} K^2(s)\,dA$$

$$\Delta A_x = \frac{\pi}{4}\,a_y\,\Delta a_x$$



Local Value $\hat{K}_y = K\,(0,\ a_y)$

Local Avg. $\bar{K}_y{}^2 = \frac{1}{\Delta A_y}\iint_{\Delta A_y} K^2(s)\,dA$

$$\Delta A_y = \frac{\pi}{4}\,a_x\,\Delta a_y$$

FIGURE 1. TWO-DEGREE-OF-FREEDOM ELLIPTICAL CORNER CRACK MODEL

ELLIPTICAL CRACK, SURFACE FLAW, AND FLAWS AT FASTENER HOLES

by

Ramesh C. Shah

Boeing Aerospace Company
Seattle, Washington

This paper summarizes the work done on evaluations of elastic stress intensity factors of three-dimensional crack problems such as embedded elliptical cracks, semi-elliptical surface flaws and part-through cracks originating at fastener holes.

Stress intensity factors for an embedded elliptical crack approaching the free surface of a semi-infinite solid and subjected to a uniform pressure, and/or a linearly varying pressure are determined in a nondimensional form as a function of position around the crack periphery, crack aspect ratio, and crack distance from the free surface(1,2). Stress intensity factors are determined numerically using an alternating technique with two solutions(3). The first solution involves an elliptical crack in a solid and subjected to normal loading expressible in a polynomial of x and y(4). The second solution involves stresses in the half space due to prescribed normal and shear stresses on the surface(5). Effect of the Poisson's ratio of the material on these stress intensity factors is also investigated(6). Nondimensional stress intensity factors for the elliptical crack in a semi-infinite solid subjected to uniform tension are compared with those derived by Nisitani and Murakami(7) and agreement between the two is excellent. Nondimensional stress intensity factors for an elliptical crack in a finite thickness plate subjected to tensile and bending loadings are determined from the above solutions by assuming negligible coupling effects between the two free surfaces(8). These results compare very well with those evaluated by Smith(9) by the alternating technique with both stress free surfaces present.

Nondimensional stress intensity factors at the maximum crack depth of a semi-elliptical surface crack in a finite thickness plate subjected to tensile and bending loadings are estimated for various crack aspect ratios and crack depth to plate thickness ratios(1,2). These nondimensional stress intensity factors are compared with those obtained by other investigators[7,10-15] and some of these results[1,10,15,16] are used to compute fracture toughness values of static fracture tests of aluminum and titanium surface flawed specimens in tension[17]. Results show that stress intensity solutions of References[1,10,16] produce reasonably constant values of fracture toughness.

A procedure is formulated to derive approximate stress intensity factors for two embedded semi-elliptical or through-the-thickness cracks originating at open holes or at loaded or unloaded fastener filled holes[17]. The procedure involves obtaining the stress distribution at locations of two semi-elliptical cracks in an uncracked solid with the fastener hole for appropriate loading conditions and then pressurizing an elliptical crack in a solid without the fastener hole with this stress distribution. Nondimensional stress intensity factors are derived for through cracks at loaded close tolerance fastener filled holes[17], and at interference fit fastener filled holes[18], for semi-elliptical cracks at open holes and at loaded and unloaded close tolerance fastener filled holes[17], and for semi-circular cracks at interference fit fastener filled holes[19]. When the problem of two semi-elliptical cracks at an open hole is reduced to a two-dimensional crack problem of through-the-thickness cracks by letting $a/c \to \infty$, stress intensity factors agree very well with those given by Bowie[20]. Solutions of two embedded semi-elliptical cracks at fastener holes in a solid are made suitable to quarter-elliptical corner cracks at fastener holes in a finite thickness plate by applying available appropriate free surface correction factors. These results were applied to static fracture and fatigue crack propagation tests containing corner cracks at fastener holes for various fastener and loading conditions[18,18,21,22], and agreement between computed and actual results is very good.

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AN ASSESSMENT OF NEAR TIP MODELING FOR 2D AND 3D CRACK PROBLEMS

by

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There are several approaches to modeling the singular field at the crack tip via the displacement approach. For purposes of discussion, these may be grouped into three categories.

In the first category, the "entire" singularity, i.e., the radial and circumferential dependence of the singular fields are embedded through the use of special elements or singular elements. These elements were used by Wilson, Hilton, Hutchinson, Shih, etc. More recently, Benzley introduced the "enriched" element, which is an isoparametric element that contains both the usual interpolation functions as well as the singular functions.

The next category of special elements enforces the radial dependence of the singular field but allows the circumferential dependence to be determined by the finite-element method. Such elements were used by Tracy, Levy, Rice, etc.

In the last category, is the "quarter point" side node isoparametric element which gives the square root singularity along the edge of the "misplaced" side-nodes. These elements have been employed by Henshell and Shaw, Barsoum, Bloom, etc.

The above summarizes some of the more well-known elements that are employed to model near tip fields. Each category of elements have their advantages and disadvantages and these were discussed. The accuracy of the above elements will be compared for several standard crack configurations. Some of the author's preliminary work on modeling crack growth for through cracks were presented.

HYBRID MODELS FOR THREE-DIMENSIONAL CRACK ELEMENTS

by

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This presentation will first review the various modified variational principles in solid mechanics and the corresponding finite-element hybrid model for the development of special singular elements for linear fracture mechanics. The basic schemes for three different hybrid models are:

(1) A scheme which is based on the assumed equilibrating stress field which also satisfies the compatibility condition inside the element, and on independently assumed boundary displacements. Such model may be interpreted as either a hybrid stress model or a hybrid displacement model.

(2) A scheme which is based on assumed equilibrating stress field inside the element and independently assumed boundary displacements. Such model is a hybrid stress model.

(3) A scheme which is based on assumed displacement field inside the element and independently assumed boundary displacements. Such a model is a hybrid displacement model.

The applications of these three schemes to two-dimensional crack elements have been reported by Tong, Pian and Lasry (1973); by Pian, Tong and Luk (1971); and by Atluri, Kobayashi and Nakagaki (1974), respectively. It appears that the most desirable element is one formulated by the first scheme for which an imbedded crack is included in the element and only the nodal displacements are left as unknowns. In the implementation of such an element, only boundary integrations are called for. Since the boundary does not include the crack tip, the integration does not involve any singular terms. The application of the other two schemes, however, will result in the use of several elements around the tip of the crack. It is then needed to introduce the stress intensity factors as unknowns in addition to the nodal displacements. Furthermore, the evaluation of the stiffness matrices in these cases involves boundary integration with singular terms and special treatments are needed in the numerical integration process.

The main discussion will be on the extension of these three models to three-dimensional crack elements. Here, however, the first two schemes may become difficult because of their requirement of assumed equilibrating stresses. It is particularly so when the problem involves a curved line as its crack front. However, for an element with a straight crack front, it is not difficult to construct equilibrating stresses that will provide the correct near field singular behavior.

In all the formulations of hybrid elements for three-dimensional crack problems, it is generally necessary to evaluate some surface integrals that involve singular terms. Also because of the difficulty in constructing the singular solutions to satisfy both the equilibrium and compatibility conditions, it is further complicated by the need for performing volume integrations that involve singular terms.

The presentation included a brief report of results of crack tip stress distributions of graphite fiber reinforced composite obtained by using hybrid stress elements, which do not include stress singularities. These include the use of laminated plate elements which take into account interlaminate shear stresses and three-dimensional solid elements. A key feature for the success of all these elements is the inclusion of the stress free conditions in the element development. Such conditions can be easily accomplished by the assumed stress hybrid method. The analysis includes the modeling of a 90/0/0/90 laminate with subcritical splits propagating parallel to the fibers of each ply. For laminated composites the near field stress distribution includes singular behavior due to the presence of the crack and the effect of interlaminar stresses at the free edge. Thus, it is uncertain whether special singular elements can be developed for such applications.

THE FINITE-ELEMENT ALTERNATING METHOD FOR ANALYSIS OF COMPLEX THREE-DIMENSIONAL CRACK PROBLEMS

by

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A new procedure for analysis of three-dimensional crack problems is described which makes the use of the finite-element method in conjunction with the alternating method. The procedure combines the best features of closed form elasticity solutions for crack problems and the finite-element method to provide an analysis tool which is flexible, relatively inexpensive and appears to be reasonably accurate.

The alternating procedure operates by applying stresses to the crack surface and using the known crack solutions to compute stresses in a semi-infinite solid at the locations of the surfaces of the solid in which the crack is assumed to reside. The opposite of these stresses is then applied to the surface of the body in question, freeing the surface of stress, and the finite-element method is used to compute resulting stresses at the location of the crack surface. The crack surface stresses are again removed as before, and the process is repeated until further contributions are negligible. The results of all iterations are then superimposed to obtain the crack opening displacements and the stress intensity factors.

Check cases have been analyzed which provide a comparison of results from this procedure with accepted two-dimensional results. Agreement in these cases is within three percent.

A series of results is presented for single and double, corner and embedded surface flaws near holes in flat plates subjected to uniform tension. These results are quantitatively compared with experimental results based on stress freezing and on fatigue studies of cracks having similar geometries. The results are qualitatively compared with high speed motion pictures taken of the initial slow growth occurring in plexiglass having cracks near a hole. The strengths of the procedure presented are as follows:

It is based on continuum crack solutions and gives the correct stress behavior at the crack tip.

It is flexible with respect to problem geometry.

It is cost effective in terms of computing costs. After set up, a solution uses 220 seconds of CPU time.

It is accurate in comparison with known solutions.

The weaknesses are:

It uses a polynominal fitting scheme to deal with stresses on the crack surface which may cause errors in problems other than the check cases.

It does not deal with the details of the problem of the crack front intersection with a free surface.

SESSION V
PRACTICAL PROBLEMS OF INTEREST TO GOVERNMENT AGENCIES

CHAIRMAN, M. F. KANNINEN

SESSION V
PRACTICAL PROBLEMS OF INTEREST TO GOVERNMENT AGENCIES

Session V consisted of presentations by representatives of five Government agencies. These represenitives described various three-dimensional fracture problems of high interest to their respective agencies. The government agencies who were represented in session were

Army (Watervliet, AMMRC)
Air Force (AFFDL, AFML, AFRPL, AFOSR)
NASA (Lewis Research Center)
Department of Transportation (TSC)
Oak Ridge National Laboratory (ERDA)

In general, these discussions were quite brief. A summary of each representative's presentation is given below.

Army (Watervliet, M. A. Hussain)

Geometries of flaws of interest at Watervliet are semi-circular or elliptical surface cracks that occur in gun tubes which can coalesce into larger frontal cracks. Prediction of the rate of formation of these cracks is complicated by the presence of residual stresses which are developed during the gun tubes manufacture (an auto frettaging process).

To a question on the dependence of crack propagation in gun tubes to strain rates the answer was given that it is not presently thought to be an important consideration. It was brought out that erosion of the inner gun tube surface is significant and of concern in itself but since its occurrence is largest at the biggest end of the gun tubes it is not considered to be a significant structural phenomena.

Army (AMMRC, D. M. Tracey)

AMMRC is concerned about structural and material modeling of cracks in various hardware components such as guns, missiles, jeeps, etc. These cracks typically result from fabrication, fatigue, corrosion or wear.

It is AMMRC's expressed opinion that the effects upon crack growth that have not been adequately addressed in the literature are those due to inertial loading (such as in impact problems) and residual stress. Furthermore, because of the three-dimensionality of the state of stress in many problems, mixed mode crack propagation can not be neglected. In view of greater use of composite materials, the effects of anisotropy and nonhomogeneities, (layers, coatings) are very real problems and in need of treatment. It was expressed that some of our efforts would be well spent if we would attempt solutions to these new aspects of crack problems rather than make small improvements on present ones.

It was also expressed that there exists a high degree of personalization in our present day publications and that we have not made specialized tools (computer codes) generally available. These tools are often undocumented. We should be more conscious of this lack of documentation of software and seek support to document each of our individual capabilities in order that others can make full advantage of them.

Air Force (AFFDL, H. A. Wood)

The new design philosophy instituted by the Air Force involves damage tolerance analysis, i.e., it is assumed that the structure contains flaws in its early life. It is felt that to insure safe operation of aircraft is to insure that propagation of these flaws does not become catastrophic in some time period. This time period could be the optimum service life or the service interval between inspections. Specification of new designs require that the engineer design the structure for residual strength and safe life. The procedure is to assume that at time zero there are flaws in the structure and that the size of the flaw is dependent upon one's ability to inspect the structure. Two requirements for failure are that

(1) Crack growth life exceeds some design value or
(2) Residual strength equals or falls short of some minimum value.

During the damage tolerance analysis, it is assumed that the flaws exist in the highest stress field areas and that they are orientated in the worst possible direction.

A most prevalent flaw geometry is one that manifests itself at holes in wing spars.

Areas of general interest to the Air Force are the effects upon crack growth rate of material variations, load history effects (such as variable amplitude and real time effects), and crack tip plasticity. The general three-dimensional geometries of interest are corner cracks, thru- and part-thru cracks, cracks at stress concentrations and the effects of the front and back surface upon crack propagation. Study of these flaws is made more difficult by the complex loadings that can occur (such as during load transfer in bolted joints and interference fit fasteners).

The basis of present criteria for crack growth are totally elastic and are, therefore, subject to question. In problems involving three-dimensional flaws, such as embedded cracks, correlation of the variation of K around the flaw to crack growth data is of great interest. It is felt that there is a need for a broad new basis for experimental techniques to verify K solutions for these problems.

To the question of whether the Air Force assumes that flaws exist at all holes, the answer was given that only one primary damage site is assumed. For example, on a 10-foot wing spar only one primary flaw is assumed. If necessary, depending upon the flaw size, the structure may be analyzed assuming that the flaw can influence the local load transfer in the structure.

Air Force (AFML, T. Nicholas)

The Air Force Materials Laboratory has also adopted the damage tolerant design philosophy for the design of engine components. A major difference in the problems encountered in engine components that are not encountered in air frame components is in the type of loading. In engine components, both mechanical and thermal cycles are experienced, during which time, time-dependent plastic deformations and creep can create strains as large as one to two percent. Under this load environment, cracks are believed to propagate under low cycle fatigue.

Thus far, the approach at AFML has been to use a linear fracture mechanics approach to these complex problems, i.e., it is assumed that ΔK

and the R ratio, $\sigma_{min}/\sigma_{max}$, are the important parameters (temperature is also considered). The validity of this approach, however, is very much in question.

The geometry of problems of interest to AFML (like those in the air frame structures of AFFDL) are cracks that originate at holes. Of great concern is what is the critical crack length and how long does it take for a crack to reach critical size?

The analysis technique in use at AFML as at AFFDL is linear fracture mechanics. However, it is AFML's opinion that a different approach is necessary for their problems.

In the discussions, it was pointed out that for the design of engine components there is a need for greater emphasis on developing the criteria for crack growth and modeling of crack growth. Present data in terms of ΔK and da/dN is not sufficient.

Air Force (AFRPL, R. Peeters)

Staff at AFRPL are presently funding studies in failure mechanisms, including initiation, velocity of propagation, and stability of cracks in solid propellant rocket motors. These problems are complicated by the fact that propellants often exhibit viscoelastic behavior. The analysis of these cracks is further complicated by the loading environment.

One example of the complex nature of these problems is that of cracks under ignition conditions (i.e., burning cracks). In this problem, the crack experiences very large thermal and pressure loads on its faces.

Other problem areas briefly mentioned were related to

- Development of a three-dimensional finite-element computer code with specialized capability for the analysis of cracks. This code will contain a displacement hybrid finite-element capability for both homogeneous and bi-material problems.
- Development of a constitutive theory for materials with process induced bonding states (i.e., nonlinear viscoelastic).
- The study of failures at the case/liner bond region of solid propellant rocket motors.

Air Force (AFOSR, W. J. Walker)

AFOSR is striving to develop the capability of life prediction through damage tolerance analyses. Current efforts include experimental and analytical mechanical studies and metallurgical studies to determine the proper measure of crack growth characterization, i.e., ΔK, energy release rate, etc.

In these studies, it is an accepted fact that errors will be present, therefore, it is believed that extremely exact calculations are not necessary. What is necessary, however, is that the basic phenomena are modelled correctly. One such phenomenon of interest to AFOSR is free edge behavior and in determining if free edge behavior is a driving force in crack growth.

Other problems that were said to be of interest to AFOSR pertained to

- The back face problem and the effect of methods of its analysis upon design life production
- The effect of plasticity upon crack growth
- Nonhomogeneous material behavior as encountered in bonded layer construction
- Biaxial loading effects
- The presence of multiple cracks as in ceramics.

NASA (Lewis Research Center, C. C. Chamis)

A great concern was expressed for the need of sound structural analysis methods to determine the load distribution and transfer through the structure before an analysis of the crack is attempted.

An interest was expressed in

- Embedded and thru crack geometries
- Mixed mode behavior (both for analytical methods and experimental behavior)
- Effects of residual and cyclic stresses.

It was felt that benchmark problems worthy of consideration are

- A surface crack in a finite thickness and width plate under tension and shear
- A surface crack in a thick wall cylinder or tube.

Department of Transportation (TSC, D. McConnell, E. Savage)

TSC, Transportation Systems Center, is a part of the Department of Transportation. The work at TSC is supported by a number of federal agencies including FRA and FAA.

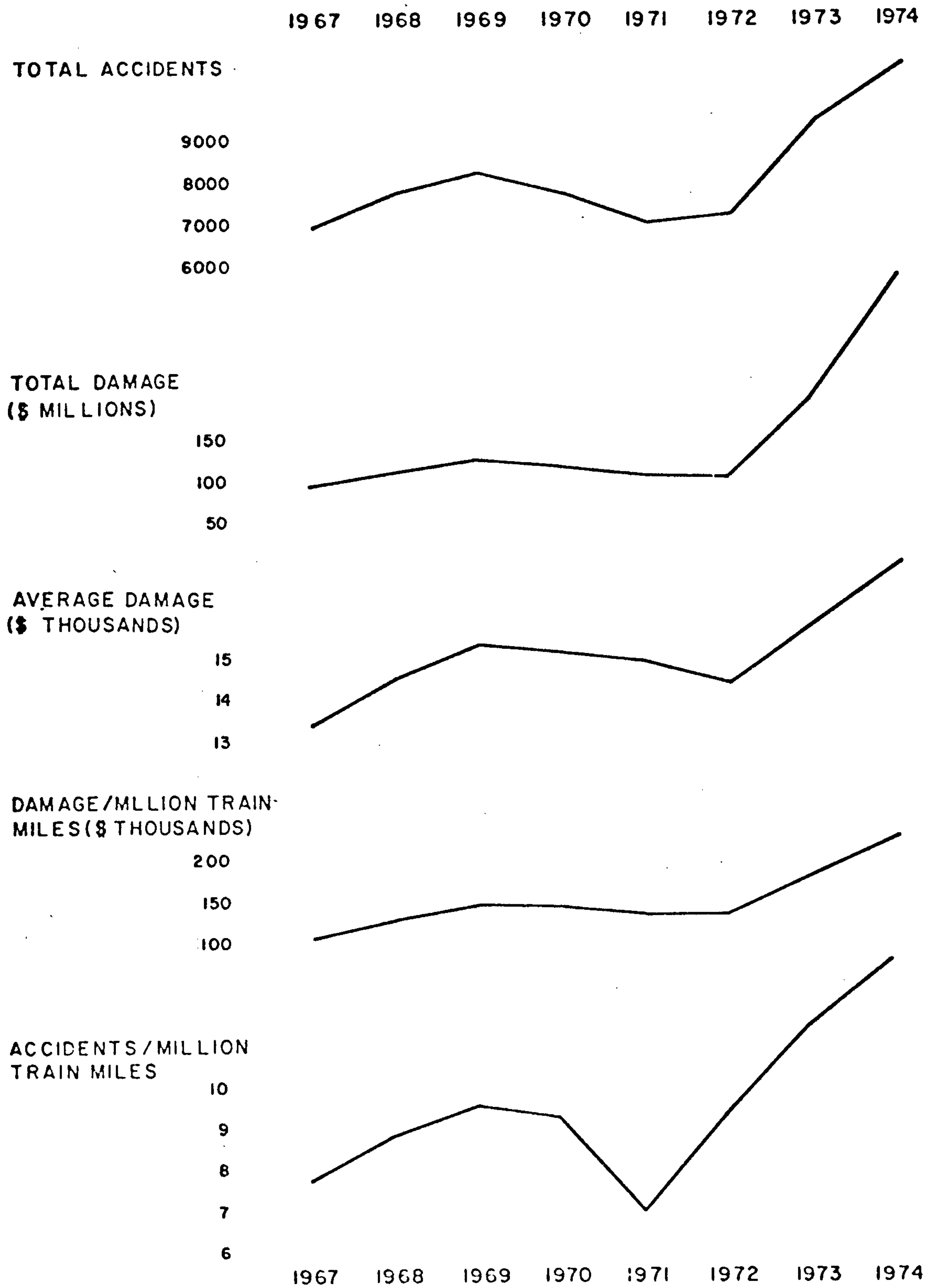
For FRA, Federal Railway Administration, TSC is involved in several problem areas involving failure of components necessary to rail transportation. These include

- Wheel failures on rail cars. This is a thermally driven problem caused by braking.
- Tank car failures. These are in the form of puncture or penetration caused by couplers during collisions of derailments. Another form of failure of interest is related to fatigue crack propagation of flaws in weldments at the head/body intersection.
- Rail failures. This is the most active area at present and a more detailed description follows.

TSC's objectives are to understand the areas of the rail subject to rail failures, determine the population of failures, determine the rate of crack growth, and the requirements for early detection. The approach that has been taken is to attempt application of present day technology to understanding the behavior of cracks in rails rather than to develop new techniques for their analysis.

Due to steadily increasing wheel loads, as a result of increase in car weight, and decrease in track maintenance, accident damage trends have steadily increased during recent years (Figure 1). This damage is primarily attributed to rail failures, (Figure 2).

Rail failures are primarily the result of formation and propagation of four types of rail flaws



FROM ACCIDENT BULLETIN NO 142, CY 1973, US DOT, FRA OFFICE OF SAFETY

FIGURE 1. TRAIN ACCIDENT DAMAGE TRENDS

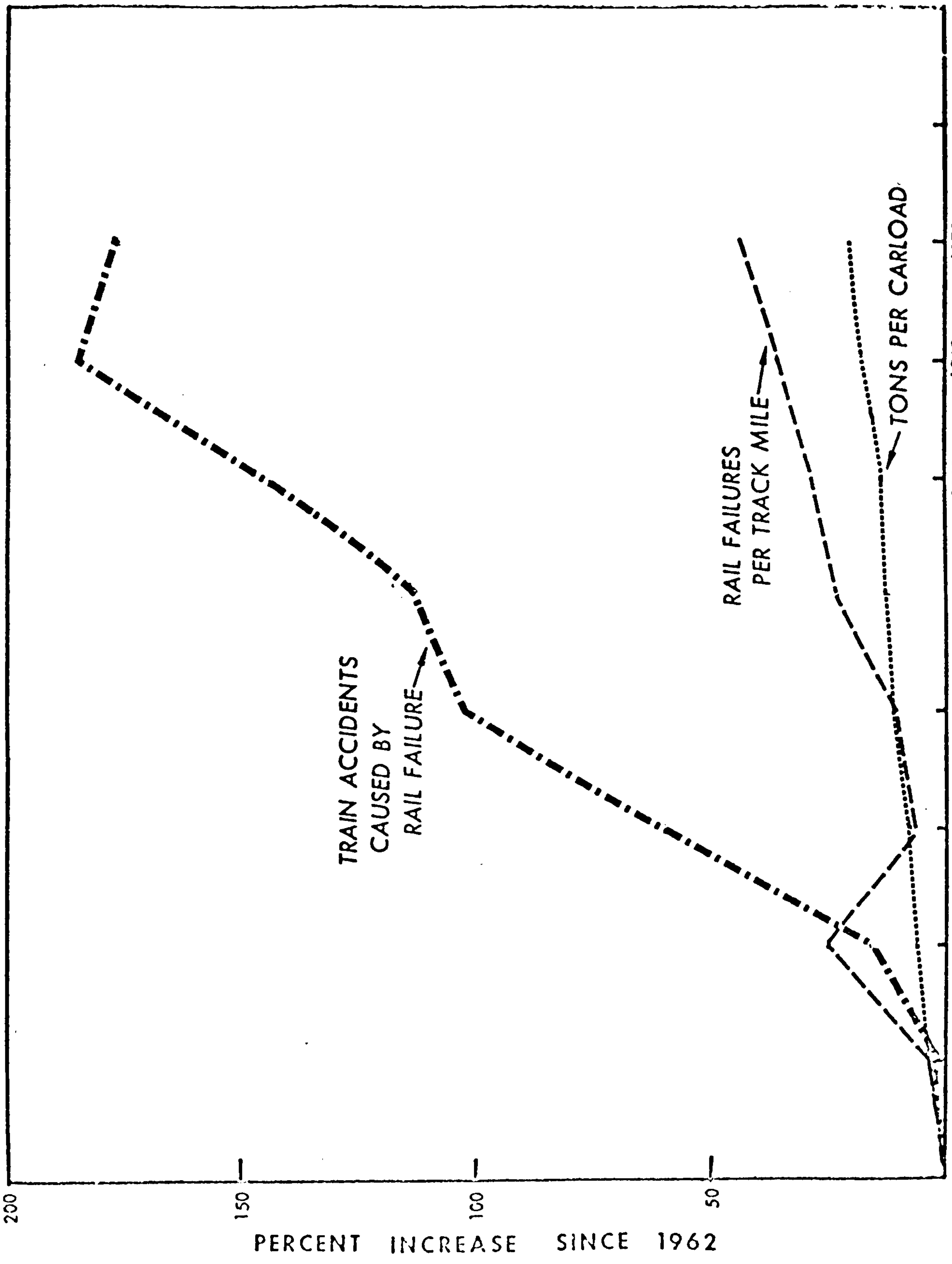

TRAIN ACCIDENTS CAUSED BY RAIL FAILURE
RAIL FAILURES PER TRACK MILE
TONS PER CARLOAD
1962
1963
1964
1965
1966
1967
1968
1969
1970
200
150
100
50
PERCENT INCREASE SINCE 1962

FIGURE 2.

- Transverse fissures (Figure 3)
- Horizontal split-heads (Figure 3)
- Vertical split-heads (Figure 3)
- Bolt hole cracks (Figure 4).

The location and shape of these flaws and recommended actions until the flawed rail is replaced are shown in Figure 5.

The transverse fissure is a vertical elliptical defect which is often totally embedded within the rail head. These flaws can cause complete rail fracture and, therefore, derailments. They are particularly difficult to detect because thermally induced compressive stresses in the rail compress the flaw, making it nearly invisible to detection techniques.

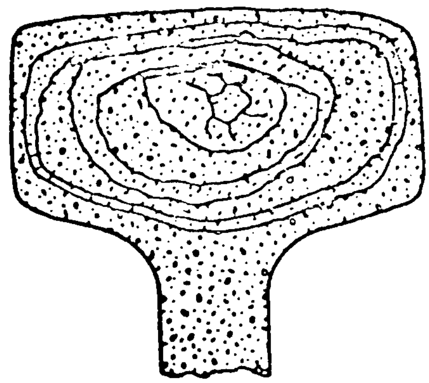
Horizontal split head defects consist initially of flaws totally embedded within the rail head; however, these flaws "break out" of the head and can grow to 3 to 4 feet in length. At this later stage in its development, for some unknown reason, the two ends of the crack can turn up and down, respectively, fracturing the rail. This also results in derailment of the vehicle.

Vertical split head defects are vertical separations totally within the rail head which causes loss of running surface.

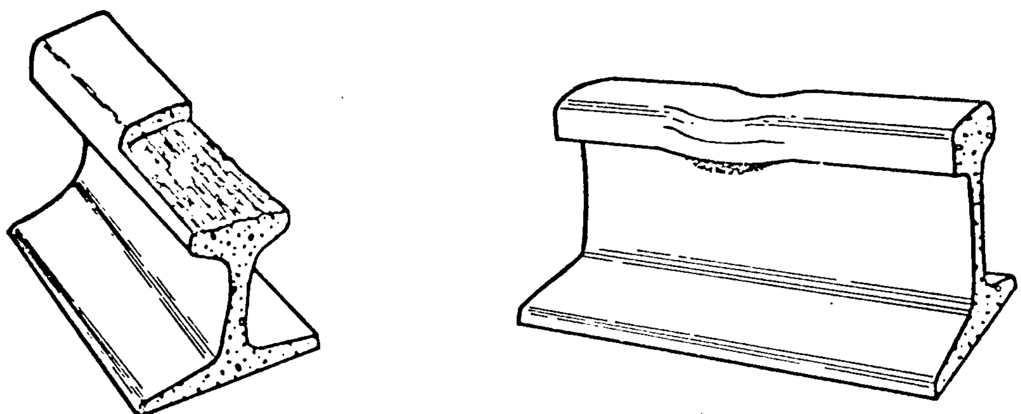
Bolt hole cracks and head web separations are flaws that initiate at bolt holes or beneath the head at the rail end at a bolted joint connection.

The loading cycles that these flaws experience are very complex, in particular, in the head region. Both vertical and lateral loads can be simultaneously applied to the rail and these loads may vary greatly from car to car. The stress distribution, Figure 6, consists, in part, of a bending stress which completely reverses during the loading cycle, and contact stresses which are highly compressive (-160,000 psi) during the loading cycle. The transverse shearing stress beneath the contact region completely reverses during the loading cycle. The problem is complicated by plastic flow that may continue over a large number of cycles. Residual stresses are known to be present in the rail head and have been measured to continuously increase after 10^6 cycles (which could be approximately 1/3 year of service).

It is thought that much of a rail's life is spent in initiation of

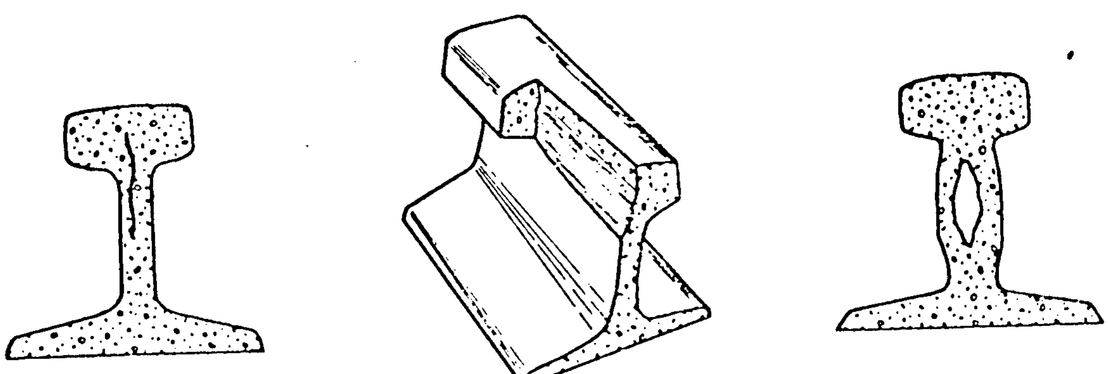


RAIL HEAD SHOWING TRANSVERSE FISSURE DEFECT



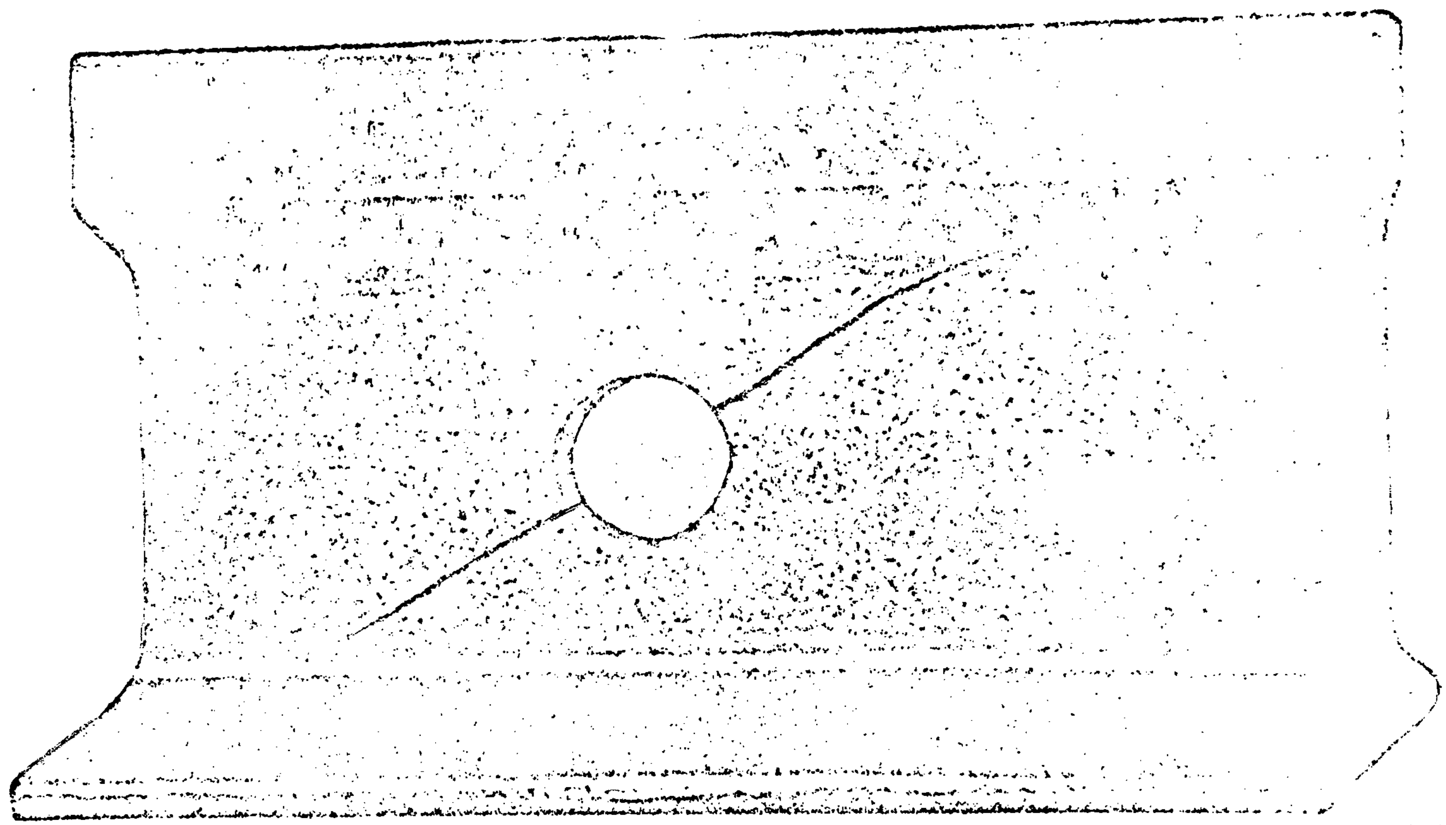
(a) (b)

HORIZONTAL SPLIT-HEAD DEFECT



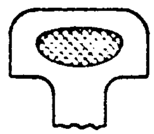
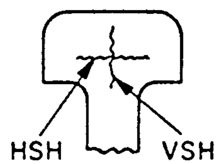
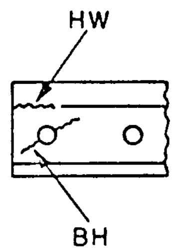
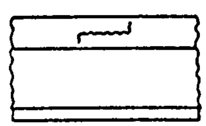
(a) split head (b) split head (c) piped rail

FIGURE 3. VERTICAL HEAD AND WEB DEFECTS



Cracks at holes are generally inclined at 45°

FIGURE 4.

DEFECT	LOCATION/SHAPE	DERAIL/Δ DETECT	SIZE	ACTION REQUIRED UNTIL DEFECTIVE RAIL IS REPLACED
TRANSVERSE FISSURE		24%/12%	HEAD AREA (a) <100%	10 mph MAX.
			(b) 100% (+)	VISUAL SUPERVISION
VERTICAL/ HORIZONTAL SPLIT HEAD		26%/18%	LENGTH (a) <2"	50 mph OR LESS*; INSPECT IN 90 DAYS
			(b) 2" TO 4"	30 mph OR LESS*; INSPECT IN 30 DAYS
			(c) >4"	10 mph MAX.
			(d) CHUNK MISSING	VISUAL SUPERVISION
BOLT HOLE/ HEAD WEB SEPARATION		17%/56%	LENGTH (a) <1/2"	50 mph OR LESS*; INSPECT IN 90 DAYS
			(b) 1/2" TO {3" (HW) / 1-1/2" (BH)}	30 mph OR LESS*; INSPECT IN 30 DAYS
			(c) > {3" (HW) / 1-1/2" (BH)}	10 mph MAX.
			(d) CHUNK MISSING	VISUAL SUPERVISION
DETAIL FRACTURE/ COMPOUND FISSURE		8%/5%	HEAD AREA (a) <20% (DF) 20% TO 100% (DF) <100% (DF)	30 mph OR LESS* UNTIL JOINT BARS ARE APPLIED AND THEN 50 mph OR LESS* 10 mph OR LESS* UNTIL JOINT BARS ARE APPLIED AND THEN 50 mph OR LESS* 10 mph MAX.
			(b) 100% (+)	VISUAL SUPERVISION

Δ SPERRY DATA, 1967–1973 AVG.
* SET BY CLASS OF TRACK

FIGURE 5.

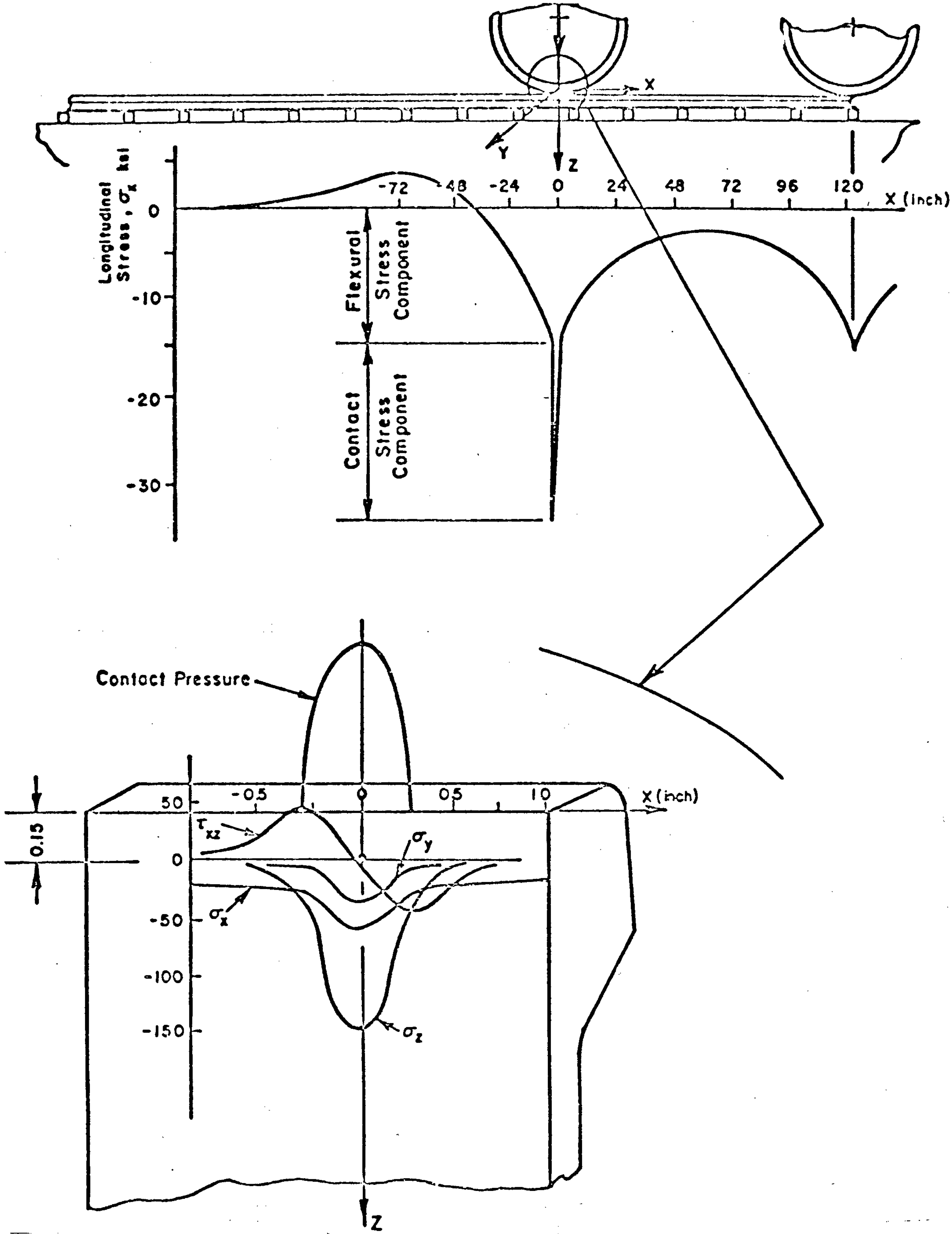

X
Y
Z
Longitudinal Stress, σ_x ksi
0
-10
-20
-30
-72
-48
-24
0
24
48
72
96
120
X (inch)
Flexural Stress Component
Contact Stress Component
Contact Pressure
50
0
-50
-100
-150
0.15
-0.5
0
0.5
1.0
X (inch)
τ_{xz}
σ_y
σ_x
σ_z
Z

FIGURE 6.

of these flaws. Once cracks have developed, it is believed by some investigators that the crack propagation rate is significantly affected by the mean value of the stress field. This is in opposition to the traditional view that fatigue crack growth is most affected by the value of K. This effect is going to be examined in ongoing research.

Naturally, since the stress states involved are generally three-dimensional, TSC is interested both in experimental behavior and analytical solutions to mixed mode crack propagation. It was also remarked that the retardation problem is also of interest.

Oak Ridge National Laboratory (ERDA, G. Smith)

Oak Ridge is presently investigating reliability of flawed pressure vessels as connected with light water reactor safety.

Reactor vessels are in general 200 inches in diameter having wall thickness of 6 to 10 inches. Operating temperature for the vessel is approximately 500 F. From a fracture point of view, two areas of interest are at nozzles and valves.

Material characteristics for the vessels are difficult to obtain due to the extremely large thickness. In order to get K_{IC} values even a 12th specimen is inadequate.

Oak Ridge has performed various model tests, such as epoxy models to examine flaws at inside corners. Numerous model pressure vessels have been also constructed. In tests involving pressure to failure typically the vessels failed at 3 times the design pressure. Flaws that were modeled in these tests were typically spherical 2 to 3 inch flaws.

SESSION VI
BENCHMARK PROBLEMS

CO-CHAIRMEN, L. E. HULBERT AND C. H. POPELAR

SESSION VI
BENCHMARK PROBLEMS

SUMMARY

In preparing for the Workshop, each of the participants were asked to suggest possible benchmark problems. A number of participants responded. These suggestions were roughly in three categories: (1) surface cracks in a slab, (2) through cracks in a plate, such as a CT specimen, (3) corner cracked holes. Task forces were set up, composed of interested participants, to consider each of these problem classes. The remaining participants discussed documentation standards for reporting benchmark problem results. The resulting documentation standards are given below. Each of the task groups reported on their deliberations to the entire group. After some discussion, benchmark problems were agreed on in each of the three categories. These consisted of problems in which ranges of parameters were optional. However, in each case, one specific geometry was chosen as the mandatory benchmark. The three geometries consist of the

(1) The surface crack (circle or semi-ellipse) in a slab

(2) The corner cracked hole

(3) The compact specimen.

Specifications for these problems are given below.

CRITERIA

I. Procedure

1. Describe method of analysis
2. Describe data reduction procedure
3. Give estimate of accuracy

II. Validation Steps

1. Two or more analyses in agreement to (?)%
2. Experimental verification

III. Results Documentation

1. K Mandatory
2. u_i Mandatory
3. σ_{ij} Desirable
4. Computer effort (in cost)
    a. Computer cost and cost algorithm
    b. Manpower required

IV. Full Problem Description

1. Boundary conditions
2. Loads
3. Geometry

BENCHMARK NUMBER 1
THE SURFACE FLAW

1. Figure 1 shows the defined geometric nonmenclature.

2. Loading shall be remote tension

   $\sigma_{zz} = \sigma_o$, $|z| = H$.

3. Material properties

   $\nu = 0.3, 0.49$

   $E = 10^7$ psi

4. Geometries

   A. Semi Circular Crack

   $\frac{a}{2c} = 0.5$, $H/W \geq 2$, $\frac{2}{t} \leq 0.2$, $\frac{W}{c} \geq 5$

   B. Part Circular Crack

   $\frac{a}{2c} = 0.25$, $\frac{a}{t} = 0.70$*

   C. Semi Elliptical Flaw

   $\frac{a}{2c} = 0.25$, $\frac{a}{t} = 0.25$, $\frac{a}{t} = 0.75$

---

* This dimension was proposed as 0.75 at the Workshop. However, a number of experimental and analytical solutions have been obtained for a/t = 0.70.

5. Presentation of Results

In accordance with documentation standards, plots and tables should be given, wherever possible of the following results:

A. $\dfrac{K}{\sigma_o \sqrt{\pi a}}$ in 10^o increments of ϕ.

Note: For the semi elliptical crack, ϕ is the "eccentric angle" measured to the point on the circle of radius a with the same x-coordinate as the point on the ellipse.

B. C.O.D. on y = 0 and x = 0.

C. Displacements on the crack surface at 10^o increments of ϕ on concentric ellipses or circles with

$$\frac{a'_i}{a} = \begin{cases} 0.1\ (0.1)\ 0.9 \\ 0.95,\ 0.98 \end{cases}$$

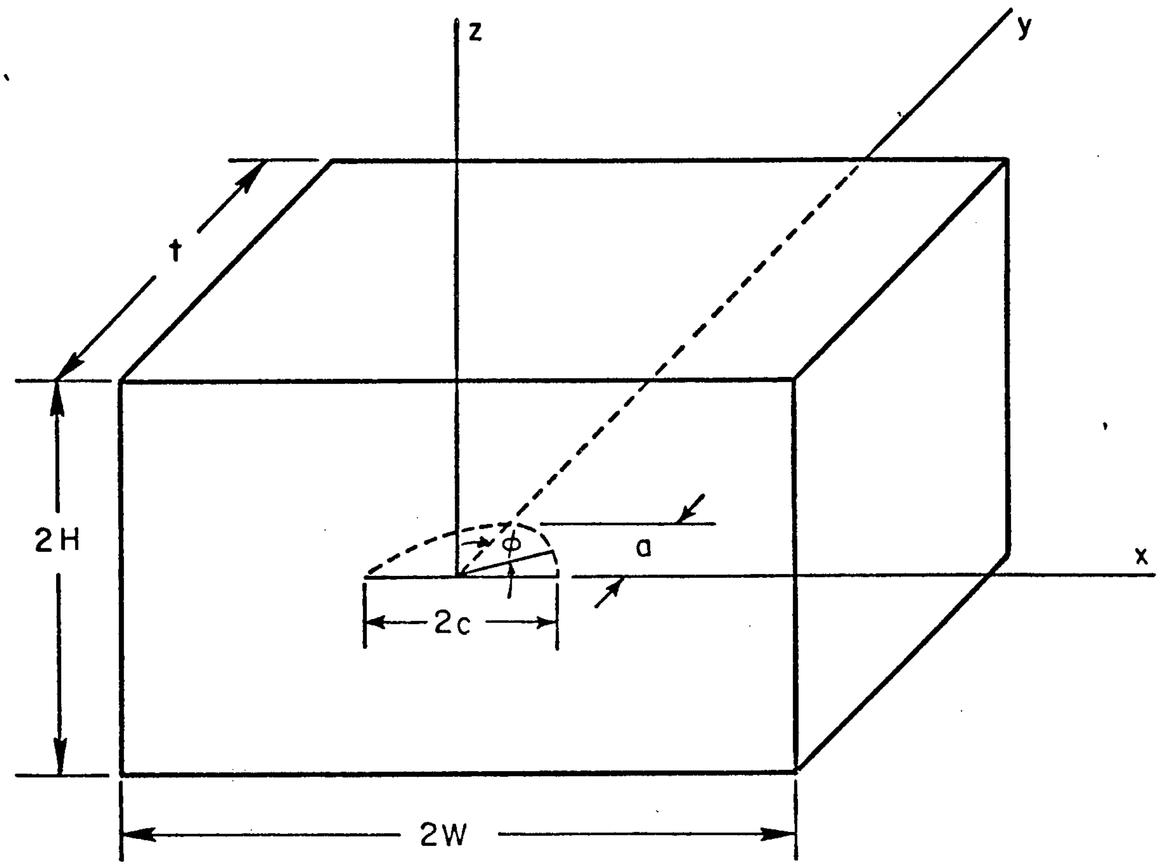


FIGURE 1. THE SURFACE FLAW

BENCHMARK NUMBER 2
THE CORNER CRACKED HOLE

1. Figure 2 illustrates the nomenclature

2. Loading shall be remote tension

   $\sigma_{zz} = \sigma_o, \ |z| = H$

3. Material properties

   $\nu = 0.3, 0.49$

   $E = 10^7$ psi

4. Geometries

   $2W = 6(2R + c) \quad H = 2W$

a/c	c/R	a/t	
0.5	0.2	0.1	
1.0	0.5	0.5	
1.5	1.0	0.75	
2.0	0.2	0.2	(Mandatory)

5. Presentation of results

   A. $\dfrac{K_I}{K_{IC}}$ for 10^o increments in ϕ

   (N.B. K_{IC} = S.I.F. at angle ϕ for an imbedded ellipse in an infinite medium for $\sigma_z^\infty = \sigma_o$)

   B. Displacement and stresses on the crack surface at 10^o increments in ϕ and at various radii position (c.f. Benchmark Number 1)

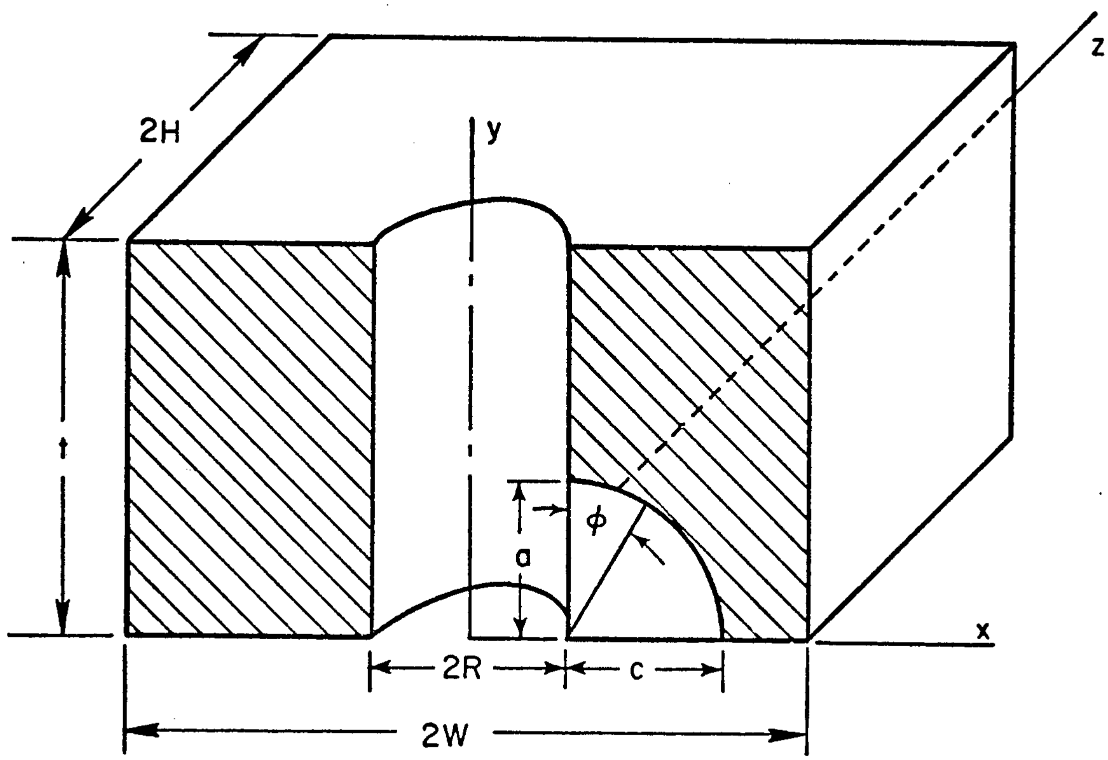


FIGURE 2. THE CORNER CRACKED HOLE

FIGURE 2. THE CORNER CRACKED HOLE

BENCHMARK NUMBER 3
THE COMPACT SPECIMEN

Considerable discussion was held on this specimen with regard to the geometry of the chevron slot machined into the specimen. In conversations with C. F. Feddersen of Battelle, who is Chairman of Subcommittee I on Fracture Test Methods of the ASTM E-24 Committee, it was established that the ASTM E399-74 standard approach to preparing a compact specimen is to machine a "convex" chevron notch with is subsequently fatigue loaded until the fatigue crack has propagated into a nearly straight through crack intersecting the surface at a distance a from the applied load. Thus, the intended test specimen contains a straight through crack and this is proposed as the standard benchmark. Possible variations on this problem include smoothly curved cracks to account for variability in the shape of the crack grown by fatigue.

Thus, the following standard benchmark is proposed:

1. Figure 3 shows the geometrical nomenclature
2. Material: $\nu = 0.3, 0.49$; $E = 10^7$ psi
3. Loading P = 1000 lb/in, through-the-thickness point loading at $x = -a = =0.5W$, $y = 0.275W$
4. Geometries
   $H = 0.6W$, $t = W/2$
   (other dimensions shown in Figure 3)
5. Presentation of results
   a. K_I as a function of z, $0 \leq z \leq t$
   b. Normal displacements and stresses for $y = 0$; $z = 0, t/4, t/2$; $-0.75W \leq x \leq 0$.

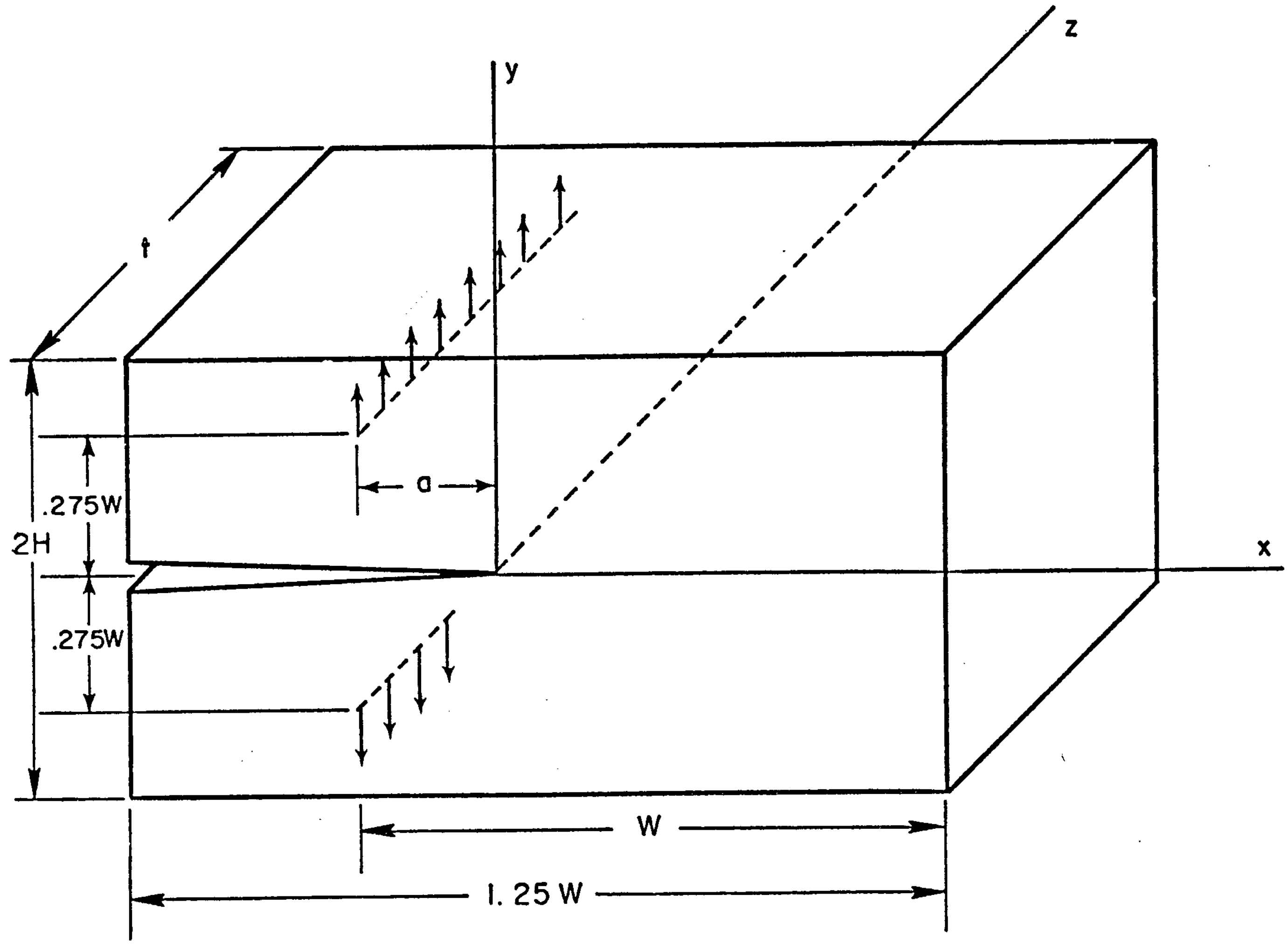


FIGURE 3. THE COMPACT SPECIMEN

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Quarter point Cracked Elements
Crack Tip Phenomena
Government Fracture Problems
Benchmark Fracture Problems

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

A three day workshop on analysis of three-dimensional stress states around cracks was held at Battelle's Columbus Laboratories on April 24-26, 1976. This workshop was attended by outstanding American scientists and government representatives concerned with three-dimensional analysis of fracture. An intensive effort was made to identify and invite all of the active researchers in this field. The result of having virtually all of the known researchers

attend the workshop presented a unique opportunity to establish the state-of-the-art in this field.

The current report includes abstracts of the papers presented, short summaries of the discussions, summaries of remarks by government representatives concerning their fracture problems, and a definition of three benchmark problems which are to be used as standards of comparisons for future analysis methods.

This workshop was jointly supported by the Air Force Office of Scientific Research and the Transportation Systems Center, Department of Transportation, with contributing support of the Army Research Office.